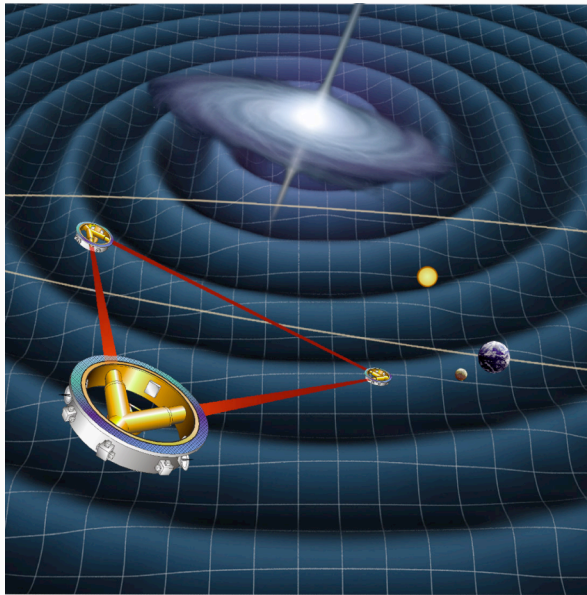


# Achieving the mid-high end of the LISA sensitivity band with the long LISA arm.



**Daniel Shaddock**

*Jet Propulsion Laboratory,  
California Institute of Technology  
Pasadena, CA 91109*

[Daniel.Shaddock@jpl.nasa.gov](mailto:Daniel.Shaddock@jpl.nasa.gov)

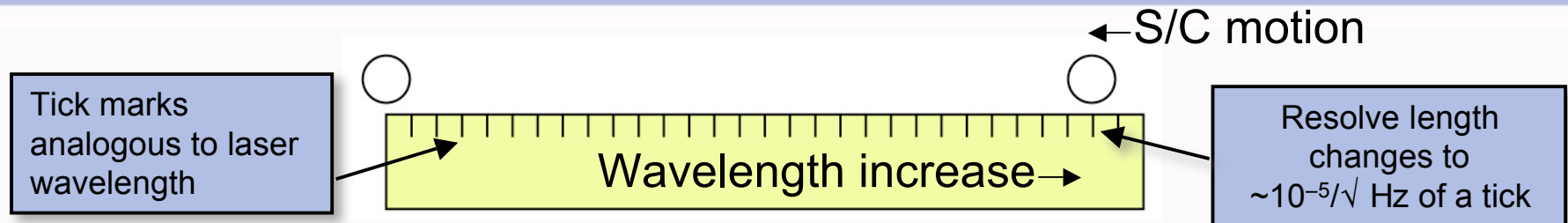


# Changes from LISA Path Finder interferometry

- Spacecraft to spacecraft interferometry not tested by LPF.
- $5 \times 10^9$  m interferometer arm lengths.
  - Key advantage of space. Long arms amplify the effect of the gravitational wave and reduce displacement sensitivity requirement.
  - Large optical loss due to beam divergence
    - Shot noise (photon counting statistics) is a significant noise source.
  - Telescope
    - Requirements are undemanding.
- Relative motion of spacecraft.
  - Arm length mismatch couples in laser frequency noise.
  - S/C relative velocity couples in clock noise.
  - Large dynamic range of phase measurement system needed.

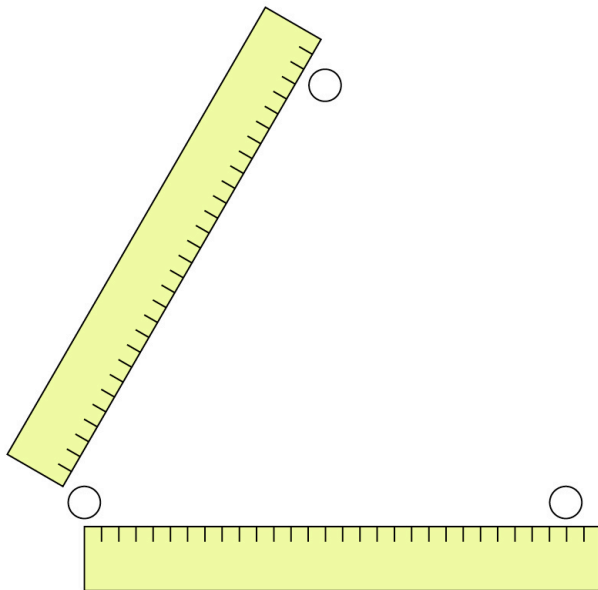


# Laser frequency noise



Spacecraft separation measured with laser 'ruler'.

Changes in the ruler length (laser wavelength) are indistinguishable from changes in spacecraft separation.



- If the difference between two equal length arms is measured, the effect of (common) ruler length changes cancels.
  - Measurement is immune to laser frequency noise (but sensitive to gravitational wave strain).
- Laser frequency noise cancellation is limited by matching of the arm lengths.



# Frequency noise in LISA

- With an interferometer arm length mismatch  $\Delta L$ , frequency noise will mimic a displacement noise,  $\delta x$ .

$$\delta x = \frac{\delta \nu}{\nu} \Delta L$$

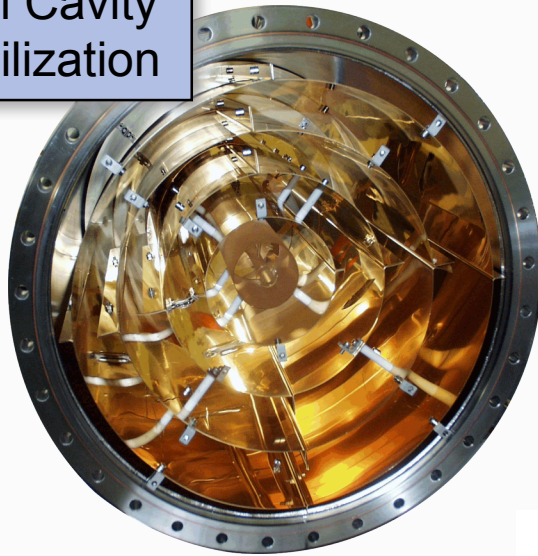
- Fractional stability after cavity pre-stabilization
  - $\delta \nu / \nu \sim 10^{-13} / \sqrt{\text{Hz}}$  @ 1 mHz with a sensitivity requirement of  $\delta x < 10 \text{ pm} / \sqrt{\text{Hz}}$
- ➡ **interferometer arm lengths must be equal to better than 100 m**
- LISA arm lengths may differ by as much as 1% or 50,000 km.  
**Cancellation of laser noise through length matching is inadequate.**

Using TDI, the **100 m arm *length mismatch*** requirement is replaced by **100 m arm *length knowledge*** requirement.

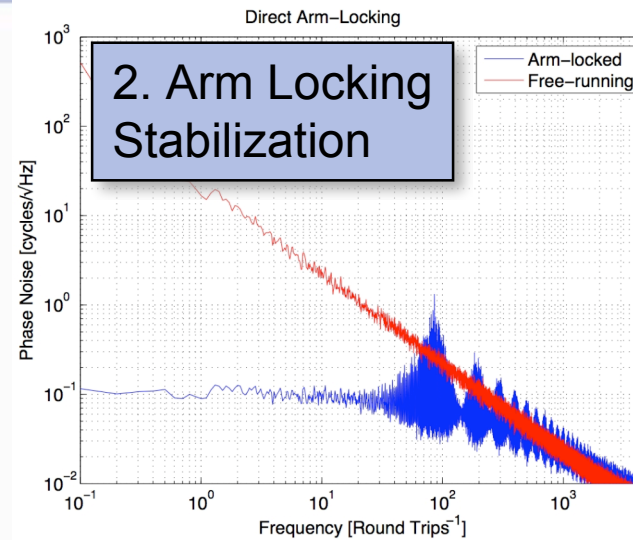


# Frequency Noise Removal

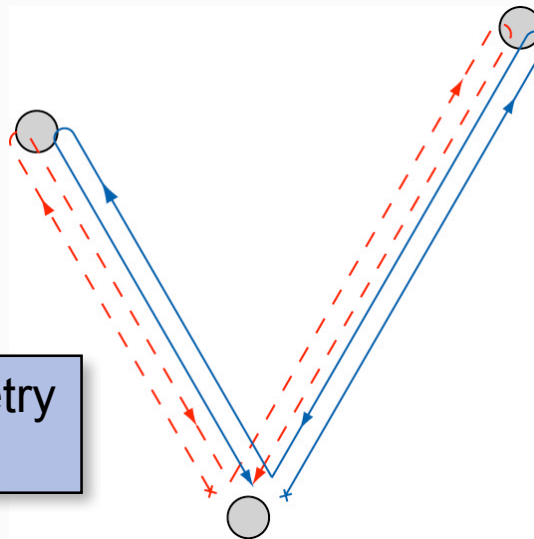
## 1. Optical Cavity Pre-stabilization



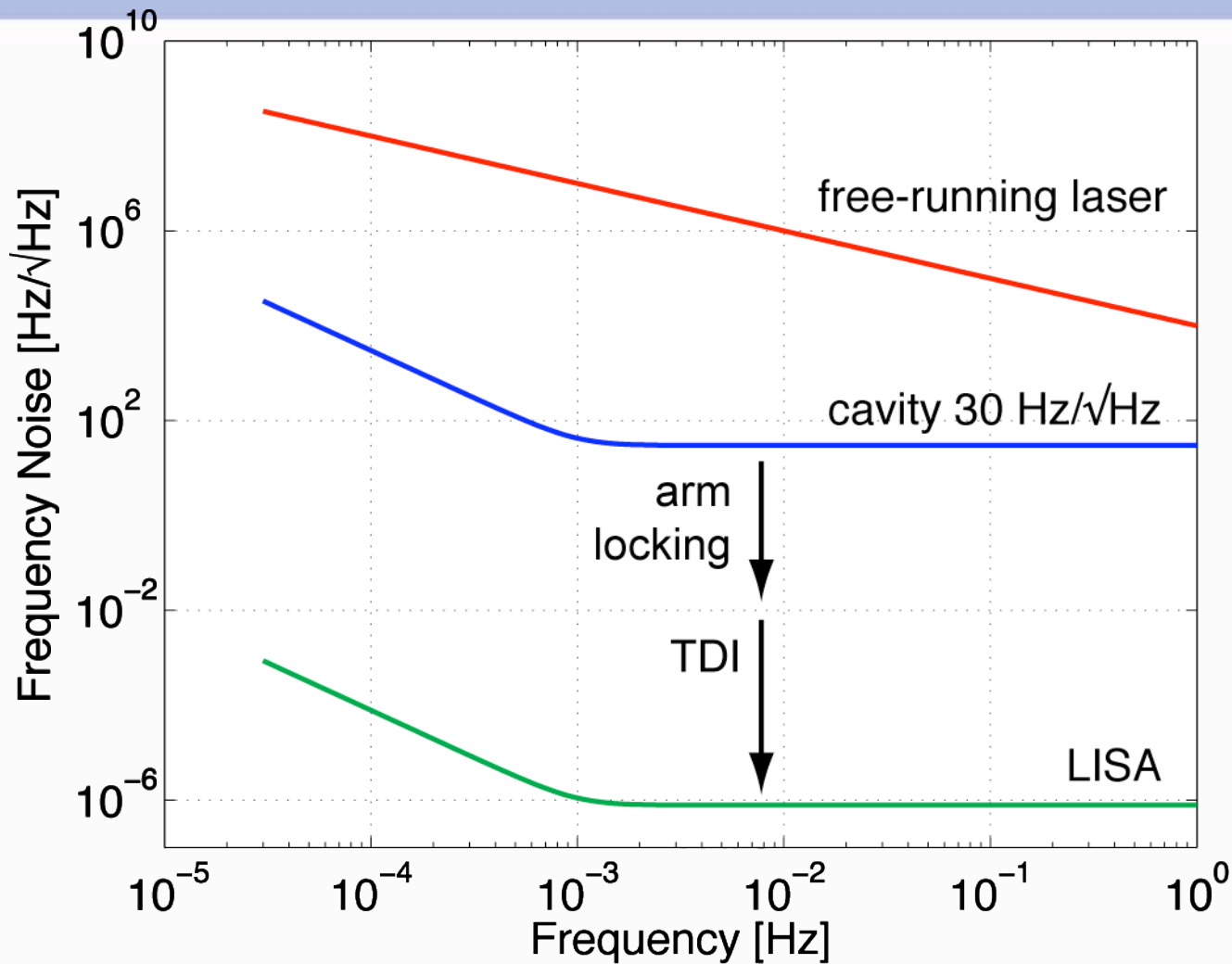
## 2. Arm Locking Stabilization



## 3. Time Delay Interferometry (Post-processing)

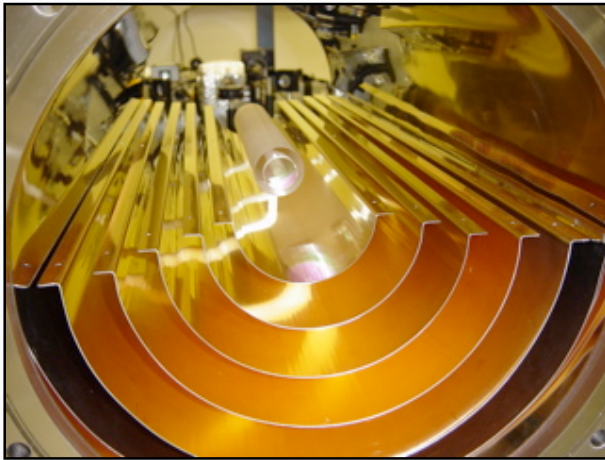


# Laser Frequency Stability



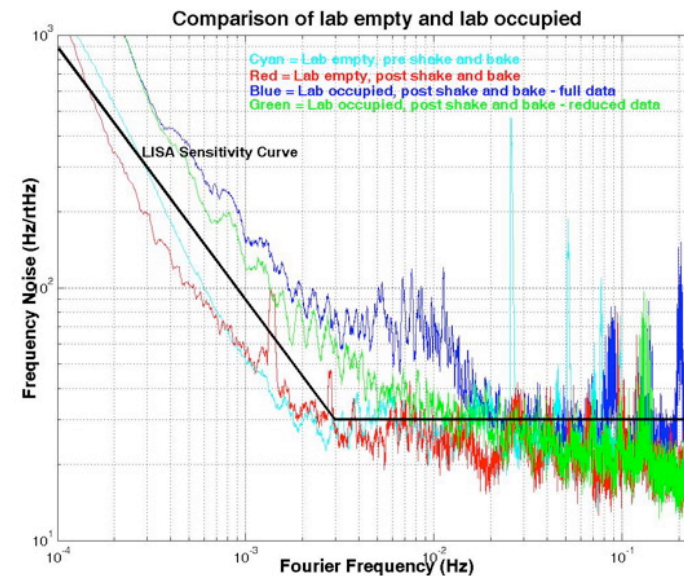
# Frequency Stabilization

Frequency stabilization system at GSFC.



- Frequency noise measured by locking two lasers to two independent cavities.
- Target frequency stability of  $\sim 30 \text{ Hz}/\sqrt{\text{Hz}}$  has been demonstrated by Mueller, McNamara, Thorpe and Camp at GSFC.

- Ultra-low expansion glass (ULE) optical cavity housed inside 5 layers of passive thermal shields.
- Temperature stability of  $\sim 10 \text{ } \mu\text{K}/\sqrt{\text{Hz}}$ .

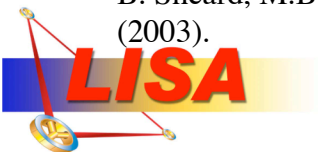


# Arm-locking

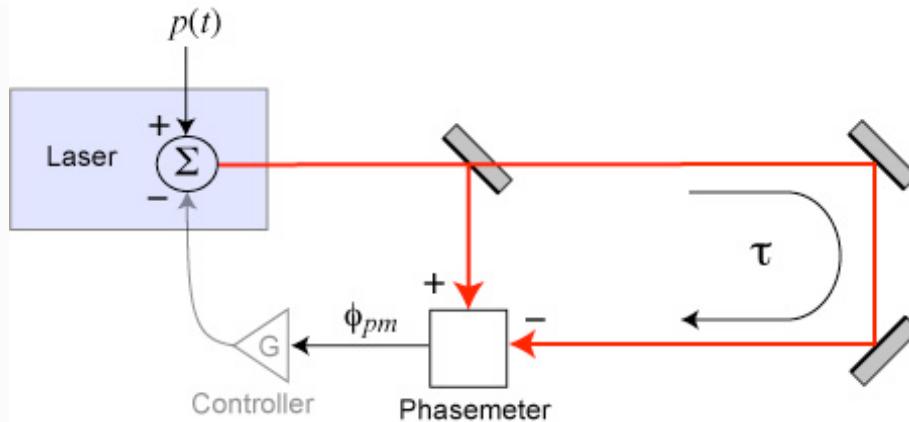
- Frequency stabilization efforts are limited by the reference cavity length stability.
- The most stable length reference available is a LISA arm:  
$$\delta L/L \sim 10^{-20} / \sqrt{\text{Hz}} \text{ (3mHz to 1Hz).}$$
- Potential problem in using arm length reference is the 33s delay
- Standard approach to dealing with delay is to limit  $f_{\text{ug}} < 1/\tau$   
Limitations:            no gain above 30mHz.  
                              low gain below 30mHz.

**Low bandwidth feedback not necessary; high gain/bandwidth can be implemented.**

B. Sheard, M.B. Gray, D.E. McClelland, D.A. Shaddock, *Laser frequency stabilisation by locking to a LISA arm*, Phys. Lett. A **320**, 9 (2003).

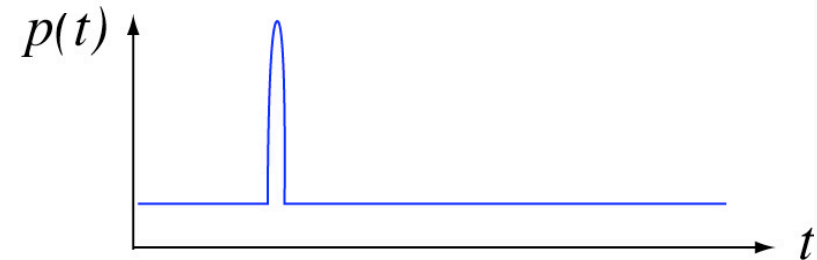


# Simple model of a LISA arm

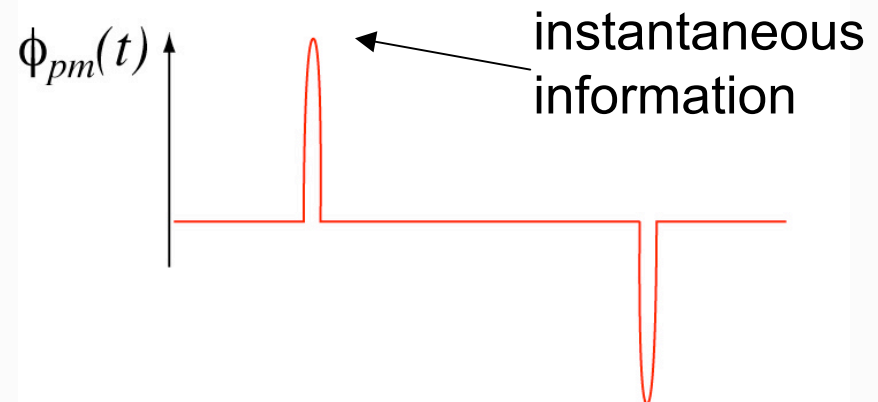


$$\phi_{pm}(t) = p(t) - p(t - \tau)$$

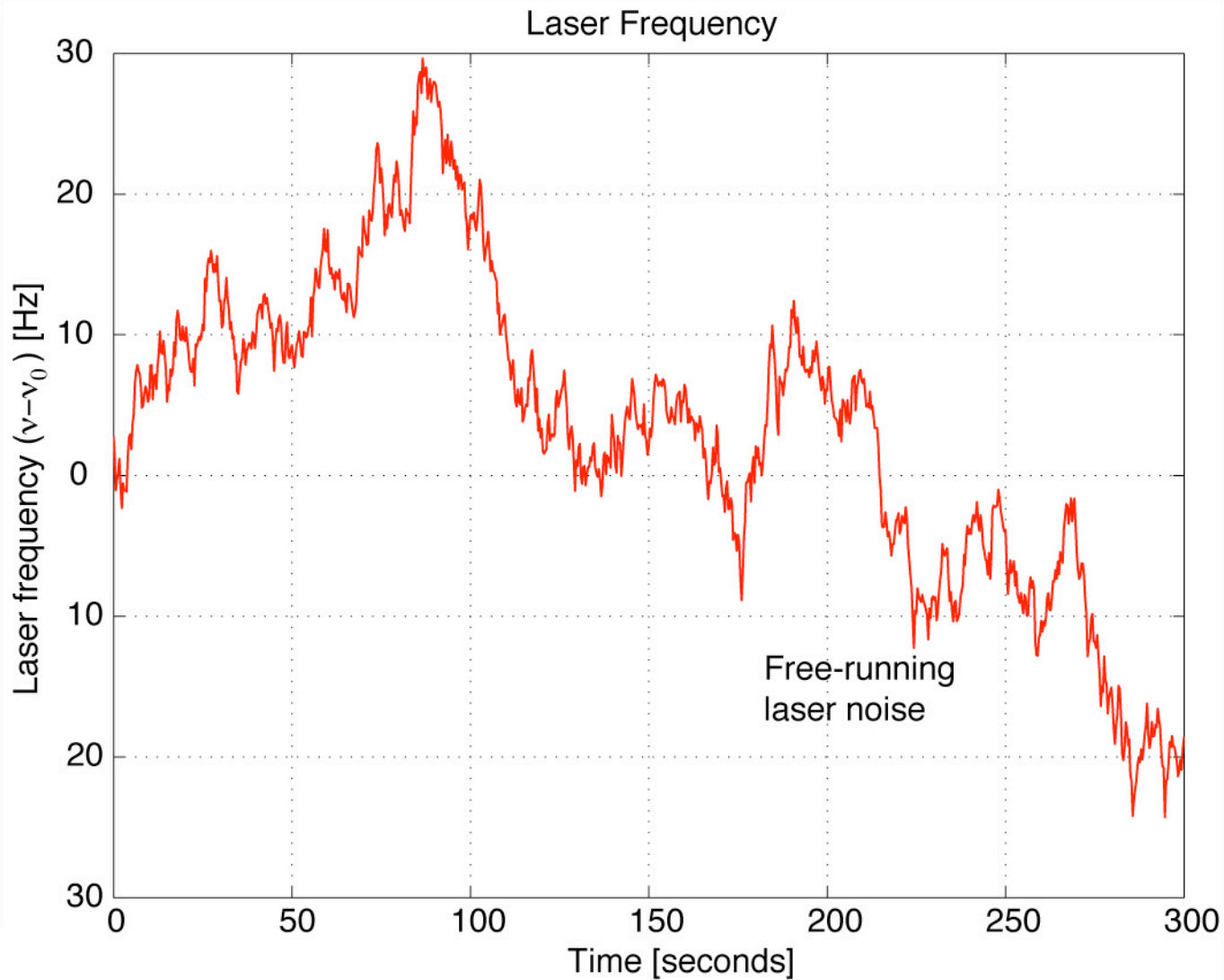
Laser phase  
impulse.



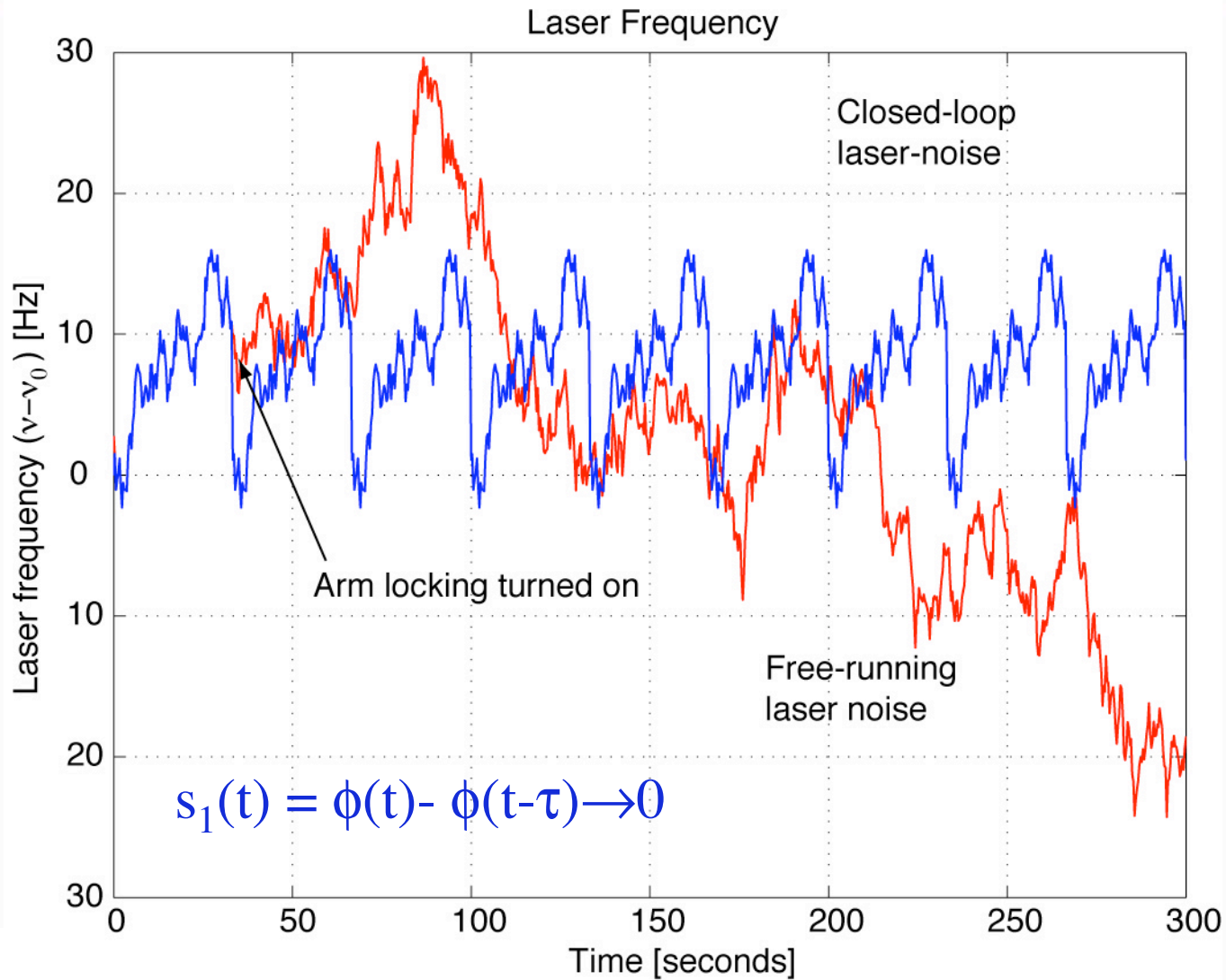
Phasemeter  
impulse response.



# Laser frequency noise

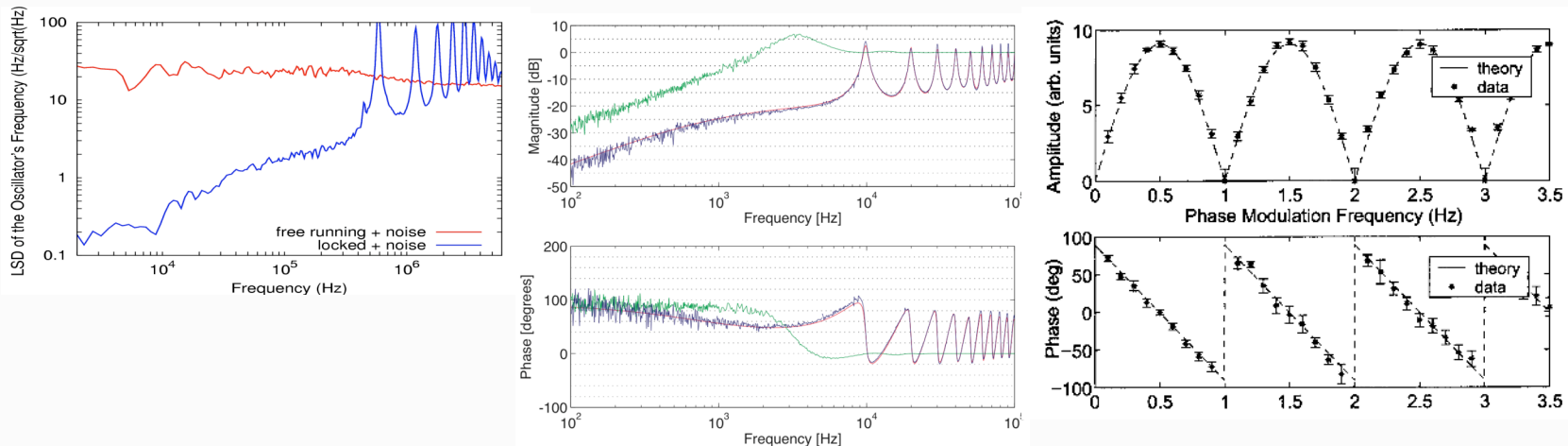


# Laser frequency noise



# Arm locking demonstrations

- Several experimental verifications
  - Electrical measurements using 300 m cable.
  - Optical measurements using 10 km optical fiber.
  - Optical measurements with up to 30 s electronic delay.



- All experiments verify analytical studies.
- The arm-locking design has been further refined to:
  1. Remove repeating noise (First 33 s noise “frozen in”)
  2. Increase in-band noise suppression.

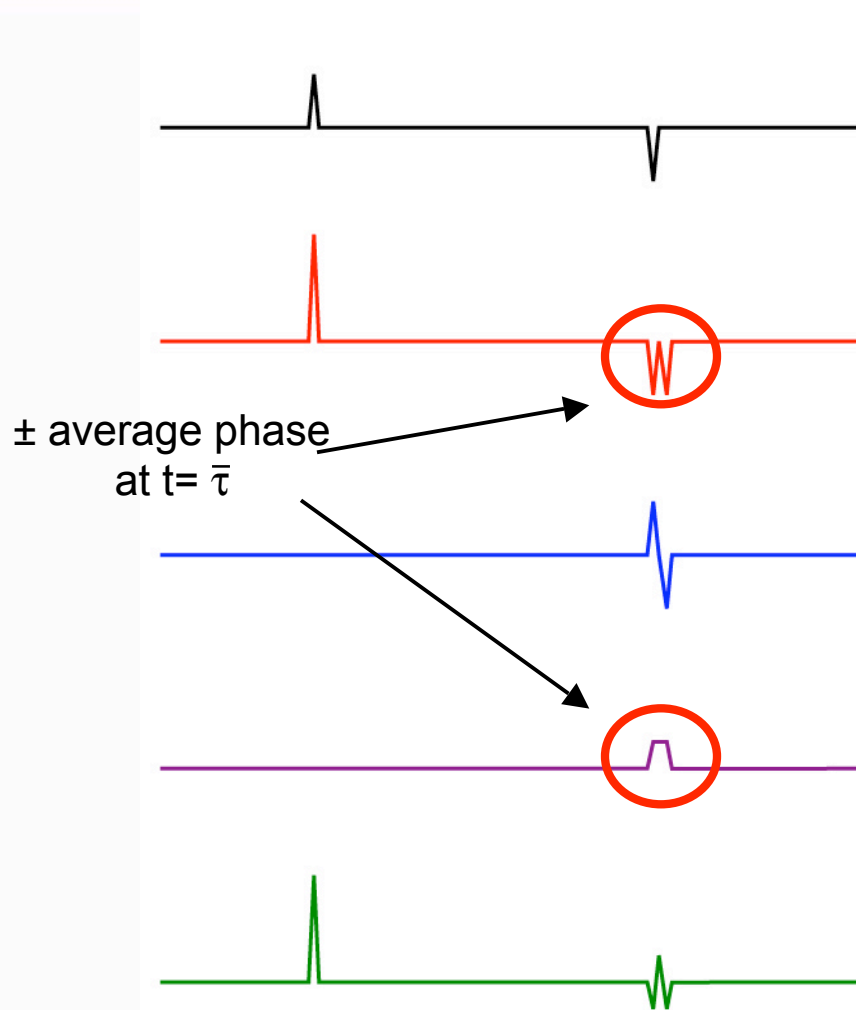


# Arm-locking

- Three goals for improving original arm-locking scheme.
  1. Reduce start-up transient noise
  2. Remove steady state “spikes” in band
  3. Increase in-band noise suppression.
- Common arm-locking discussed by WG2 in 2003
  - Potential to increase gain significantly below 3 Hz (if arm length mismatch can be guaranteed).
- New scheme to remove start-up transients proposed by Markus Herz.
  - Uses information from a second arm (assuming first arm is tightly arm-locked).
  - Uses feedforward to cancel noise.
- Direct arm-locking is a modification of Herz suggestion that uses combination of common and differential signals in feedback.
  - Uses feedback only.
  - No special “initialization” required to remove transient.



# Constructing a better sensor



Single arm

$$s_1(t) = \phi(t) - \phi(t - \tau_1)$$

Common arm

$$s_1(t) + s_2(t) = 2\phi(t) - \phi(t - \tau_1) - \phi(t - \tau_2)$$

Differential arm

$$s_1(t) - s_2(t) = \phi(t - \tau_2) - \phi(t - \tau_1)$$

Integrated difference

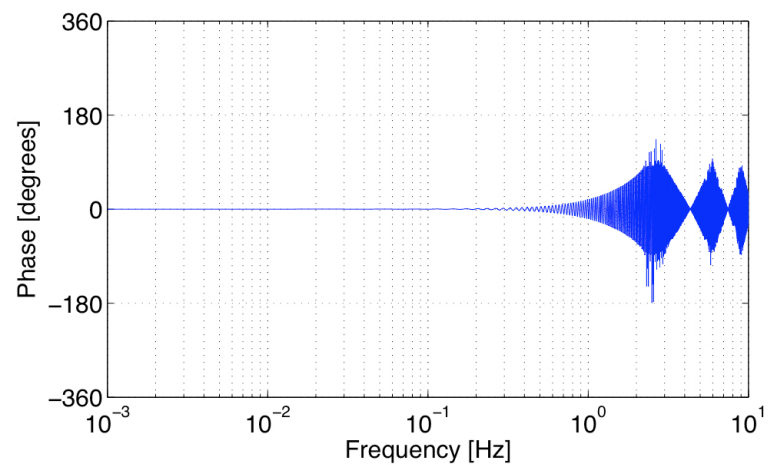
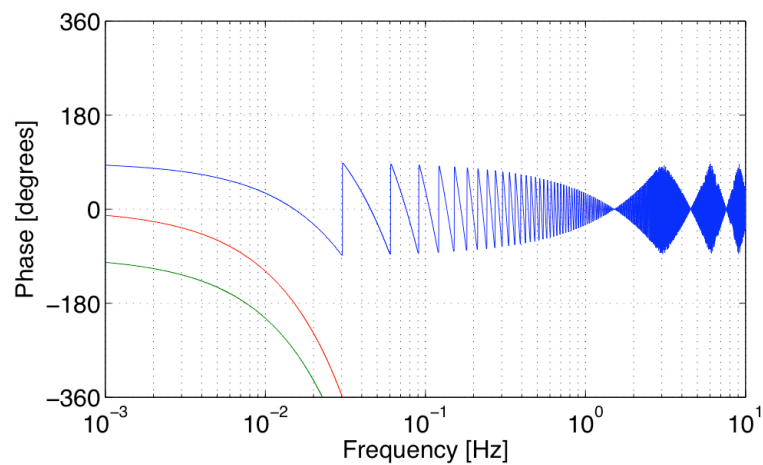
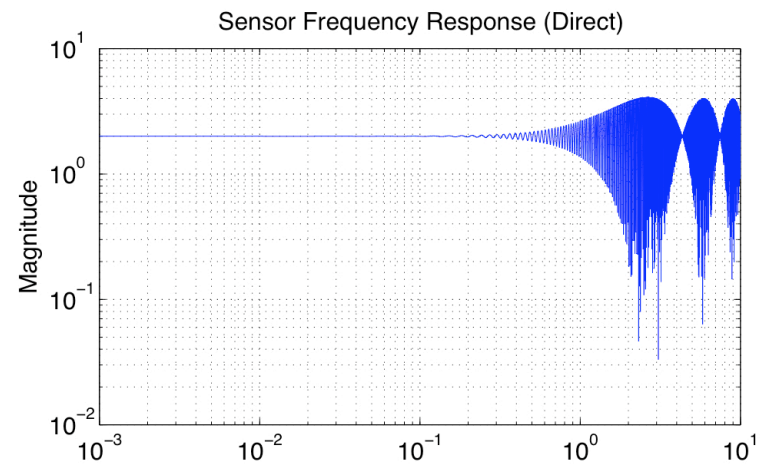
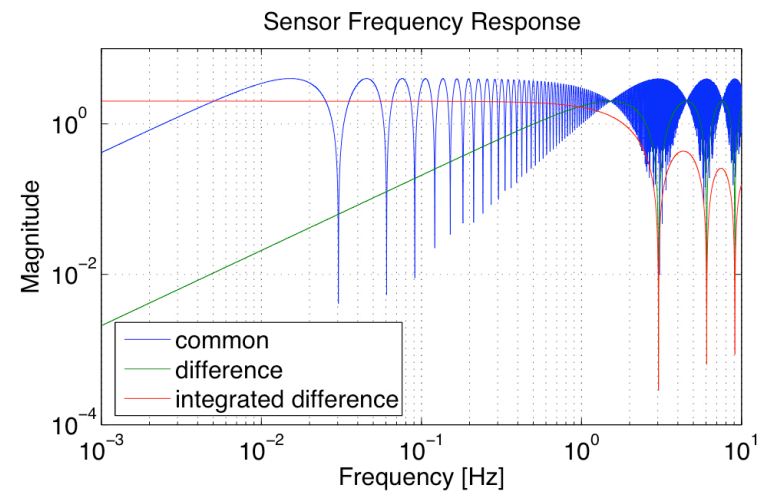
$$\int s_1(t) - s_2(t) dt$$

Direct

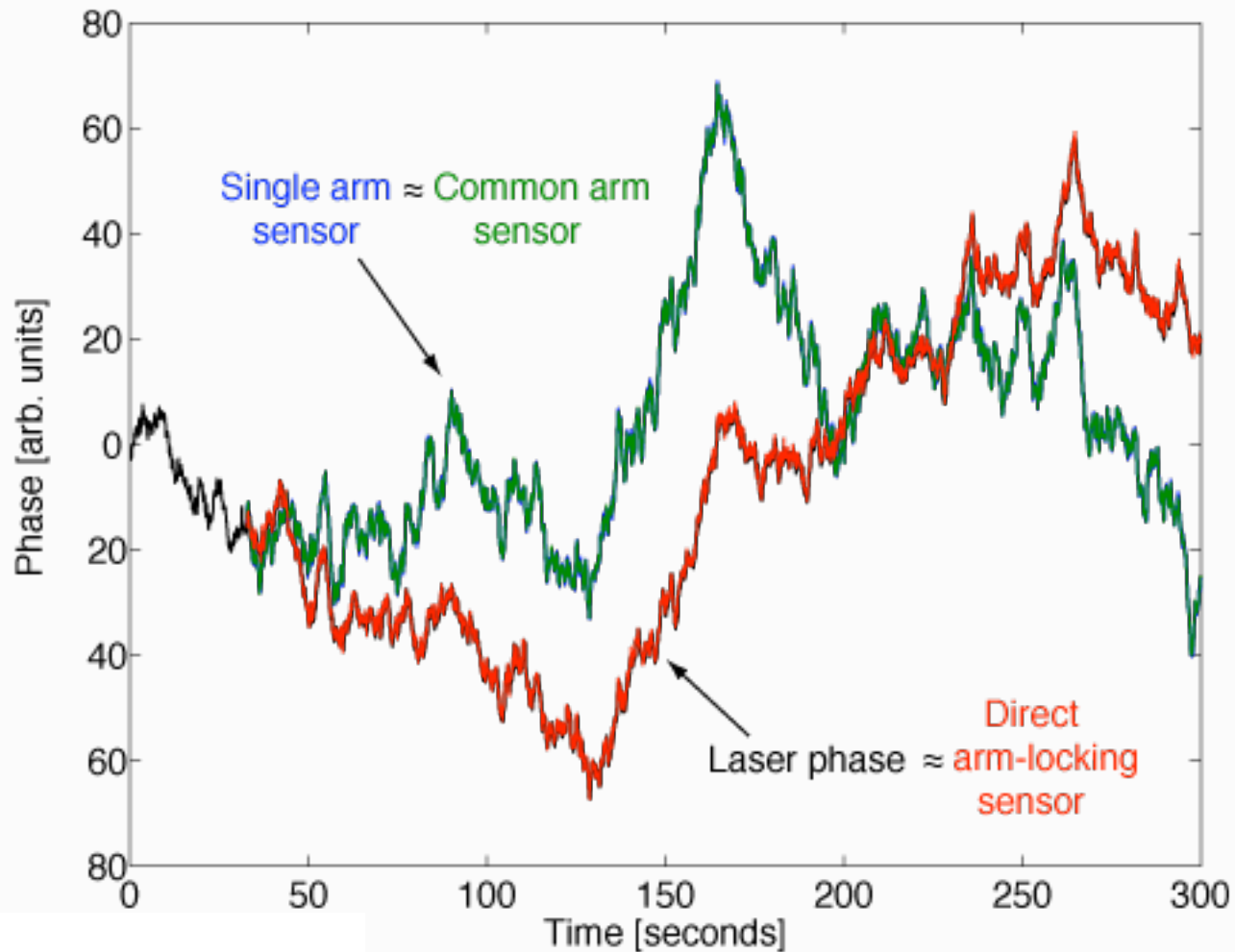
$$s_1(t) + s_2(t) + 1/\Delta\tau \int s_1(t) - s_2(t) dt$$



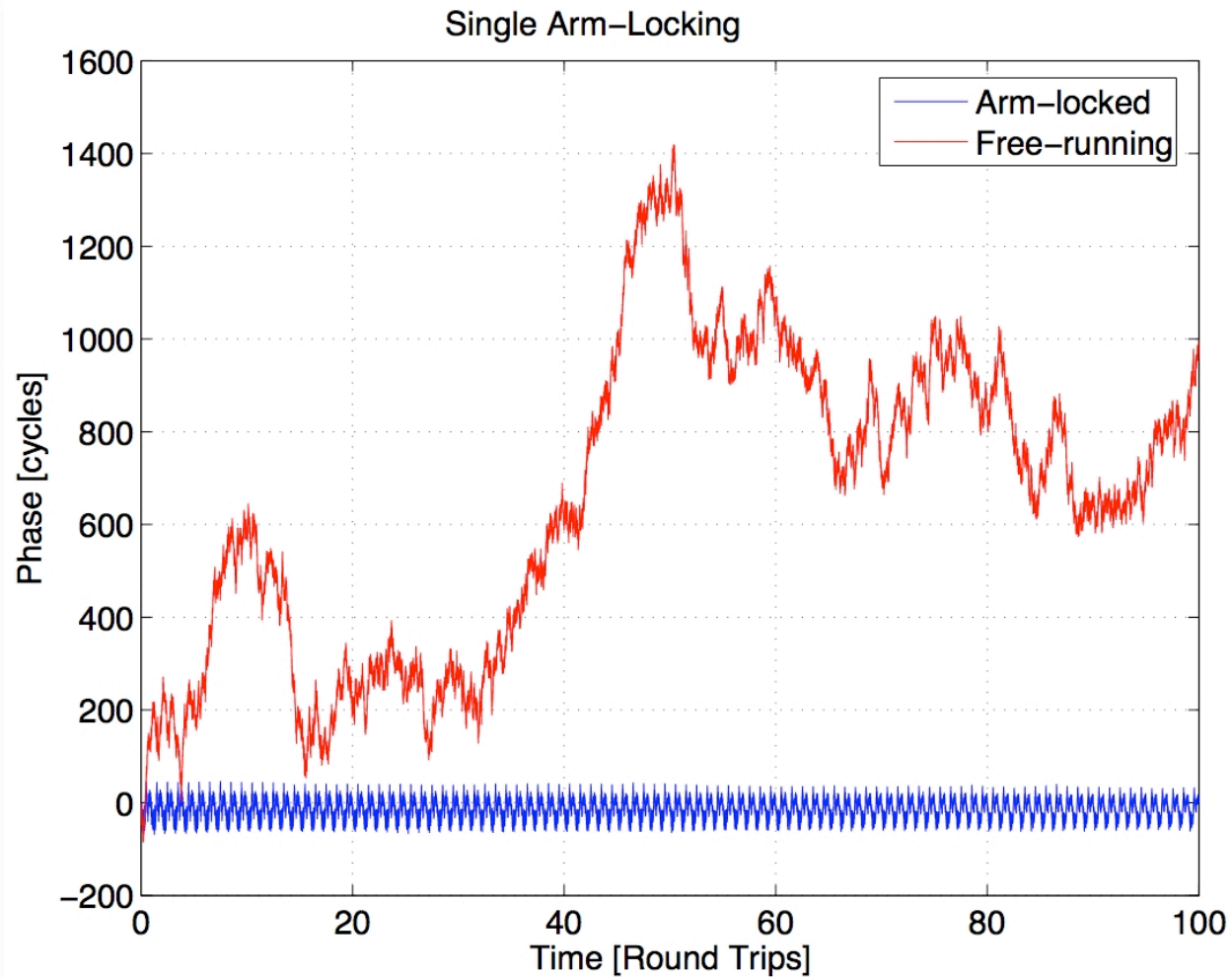
# Sensor Frequency Response



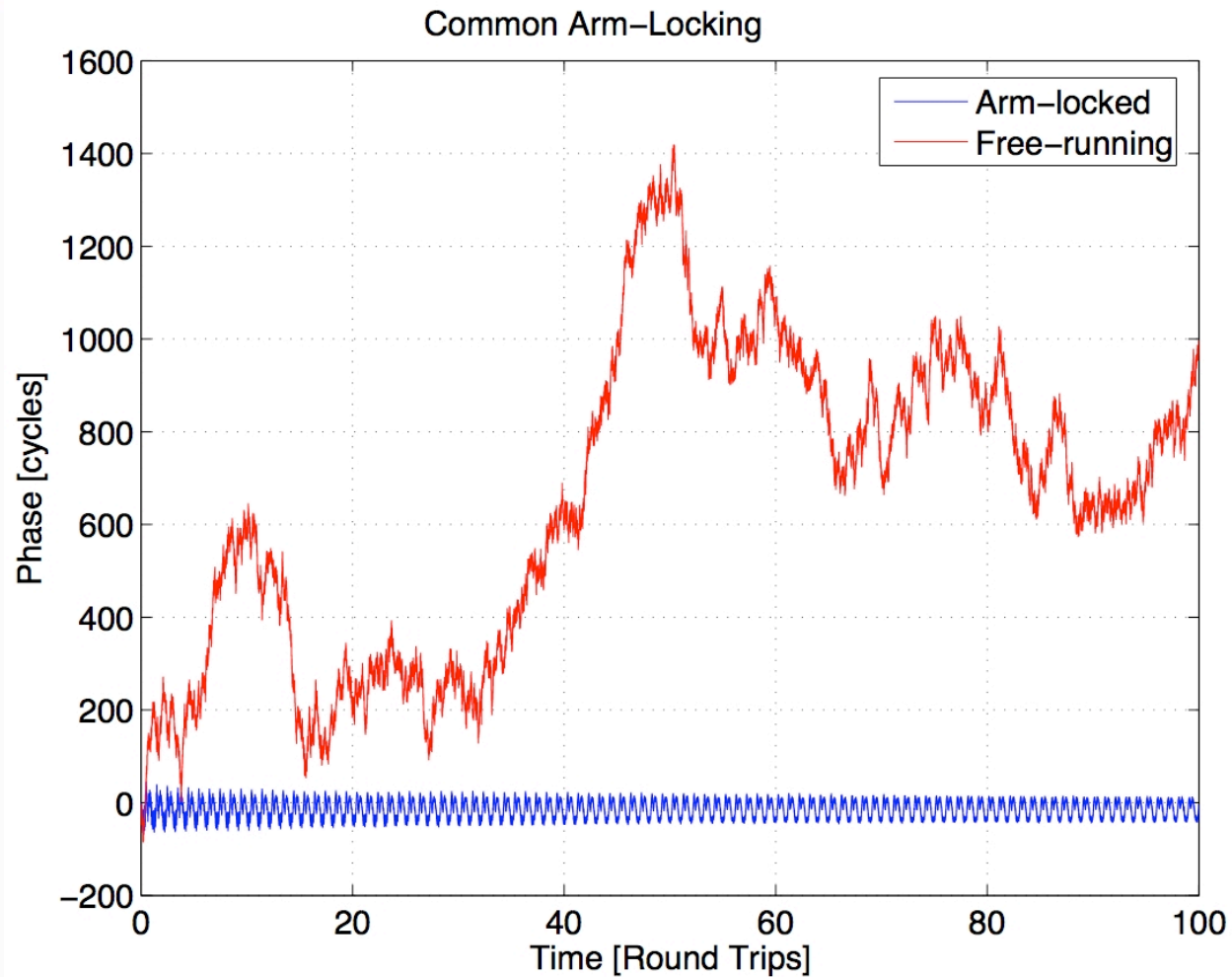
# Arm-locking sensor comparison



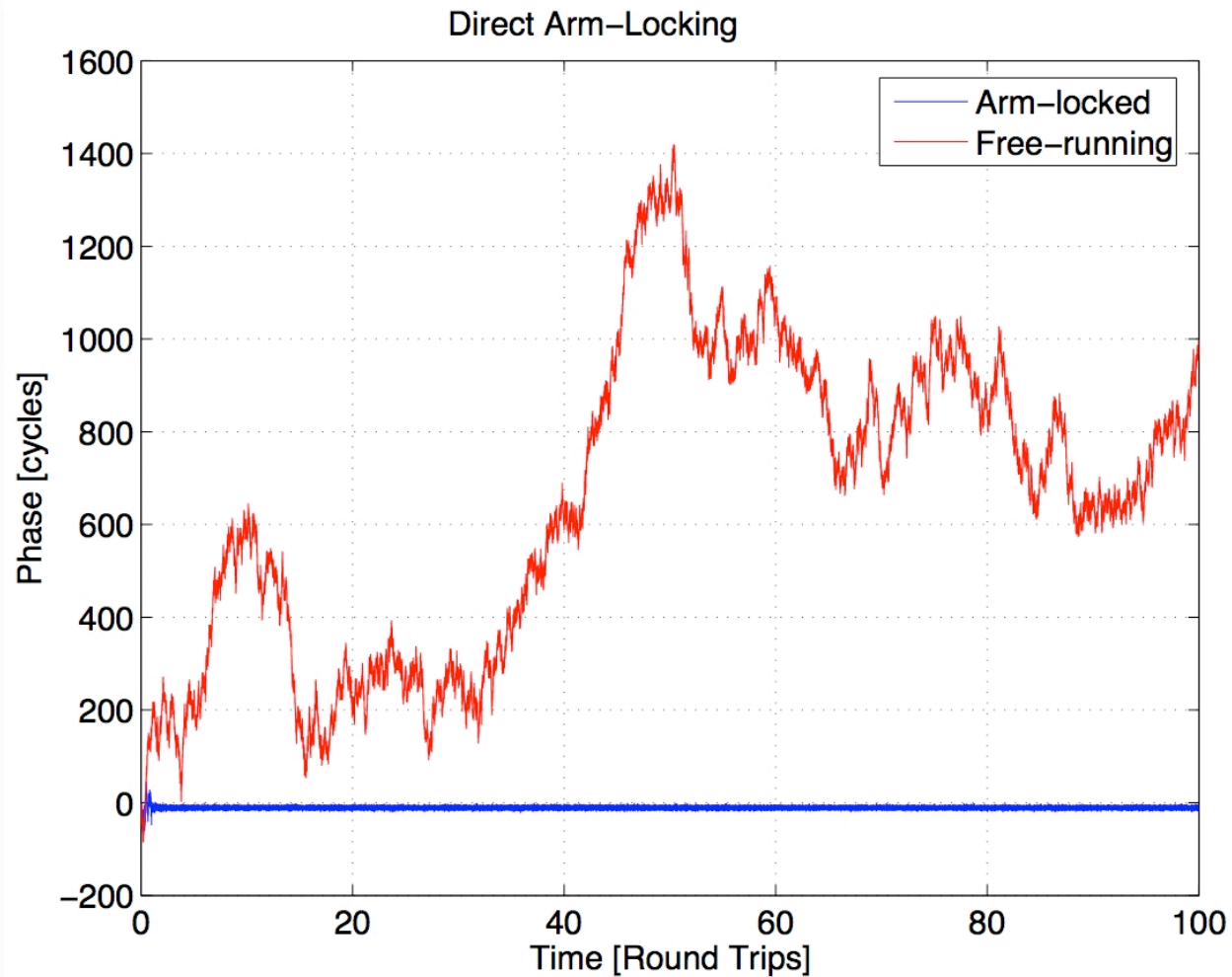
# 1. Single Arm-Locking



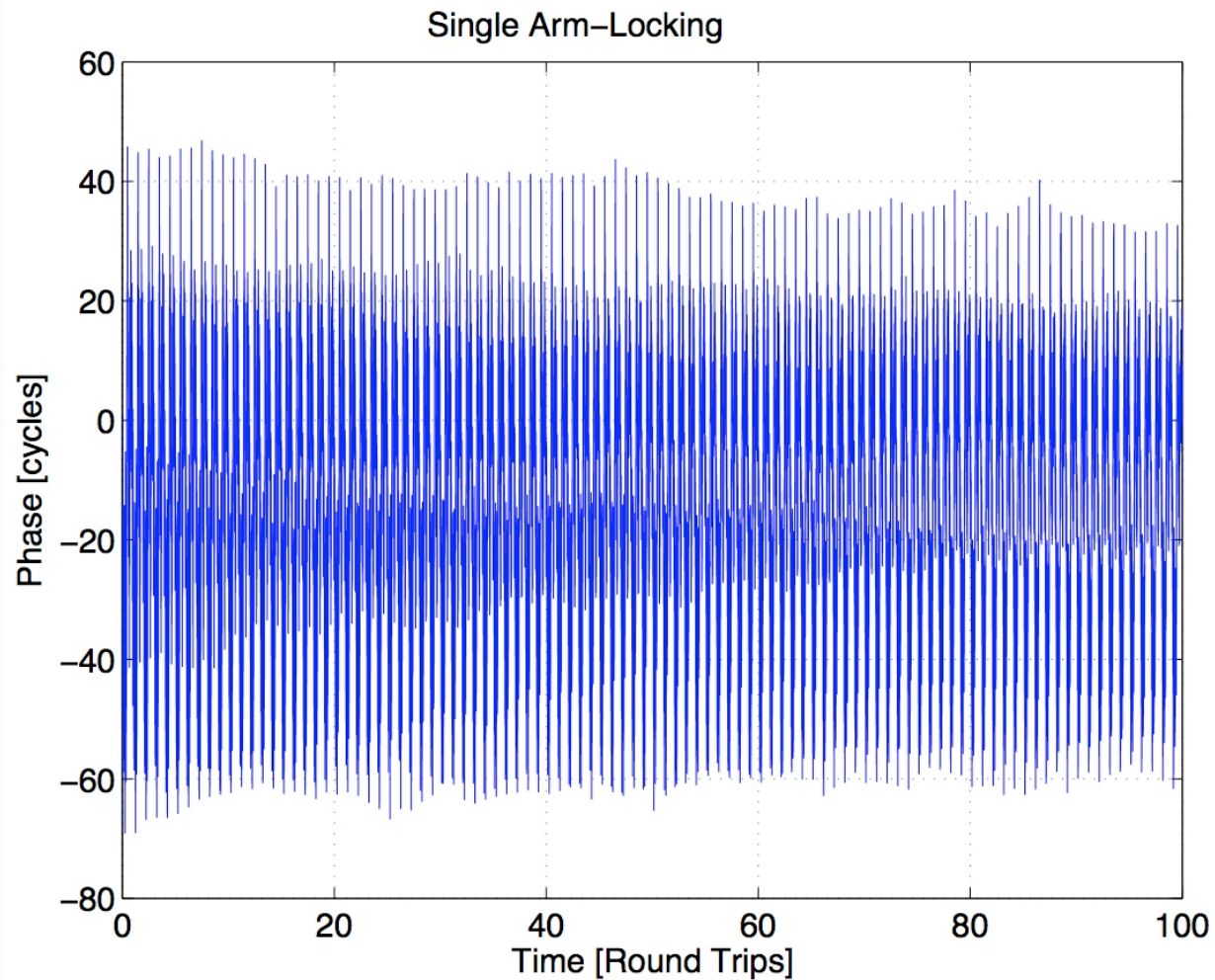
## 2. Common Arm-Locking



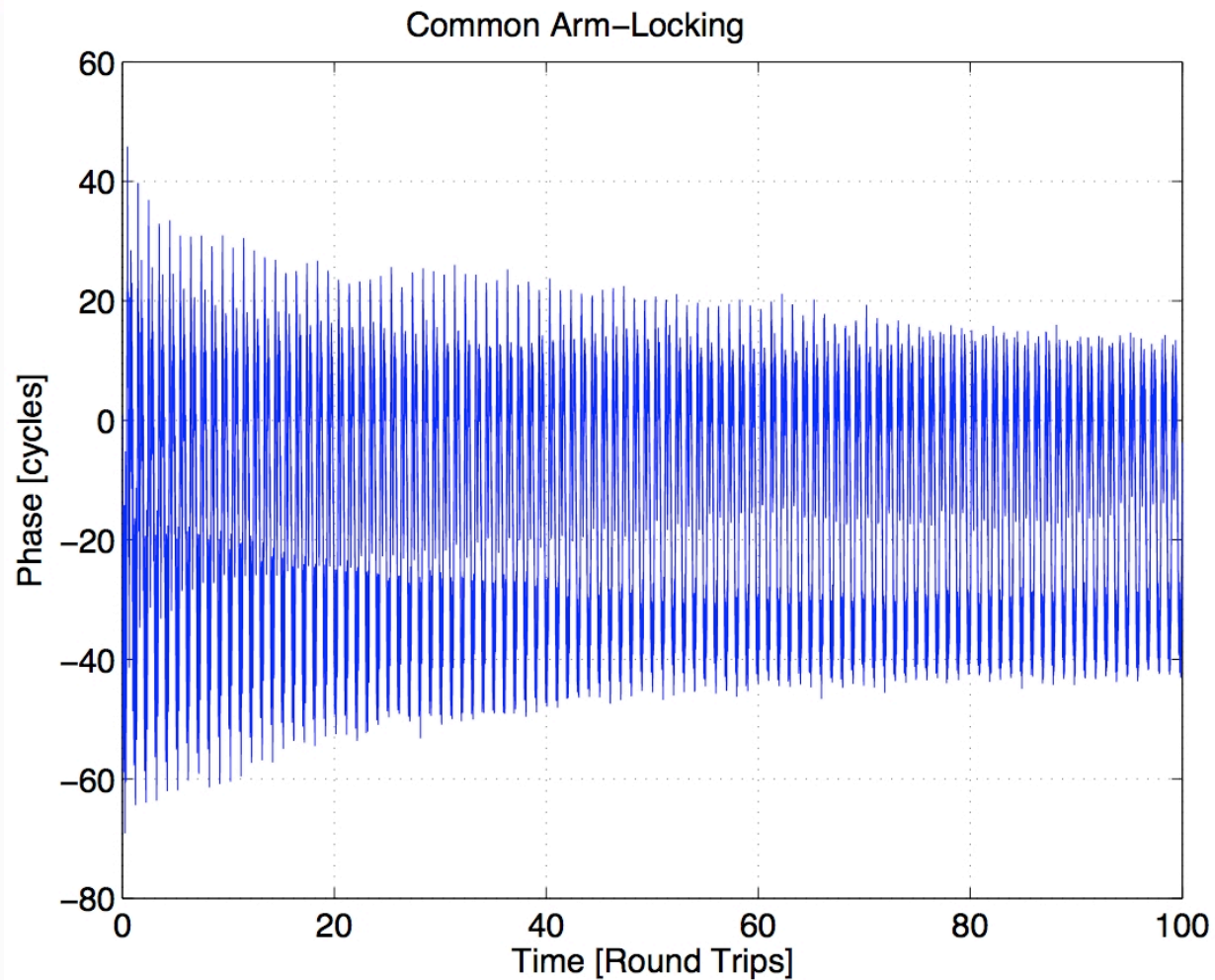
### 3. Direct Arm-Locking



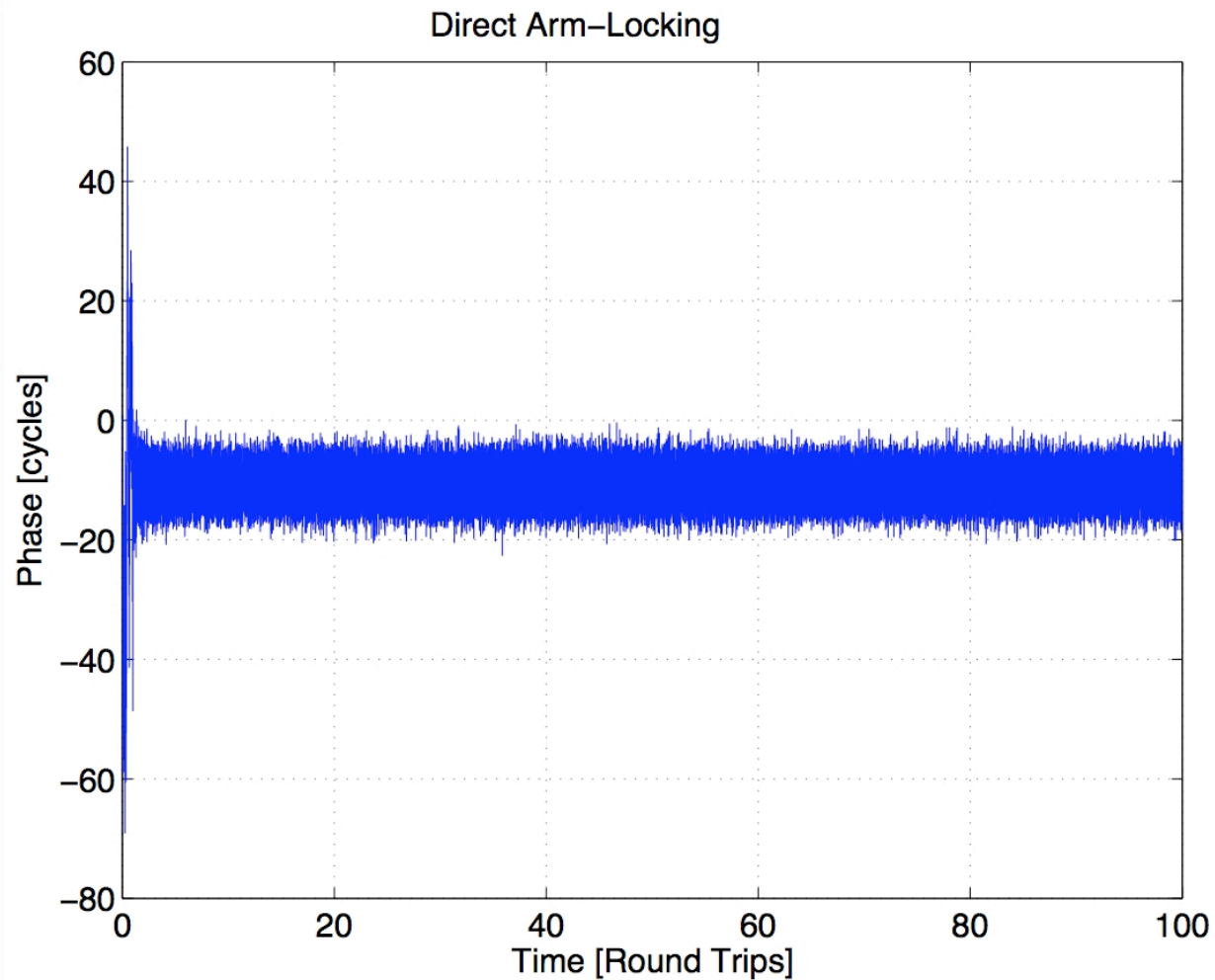
# 1. Single Arm-Locking



## 2. Common Arm-Locking

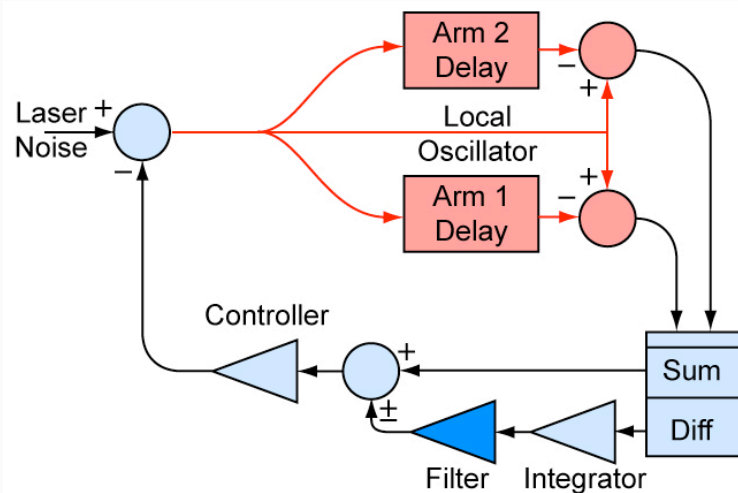


## 5. Direct Arm-Locking



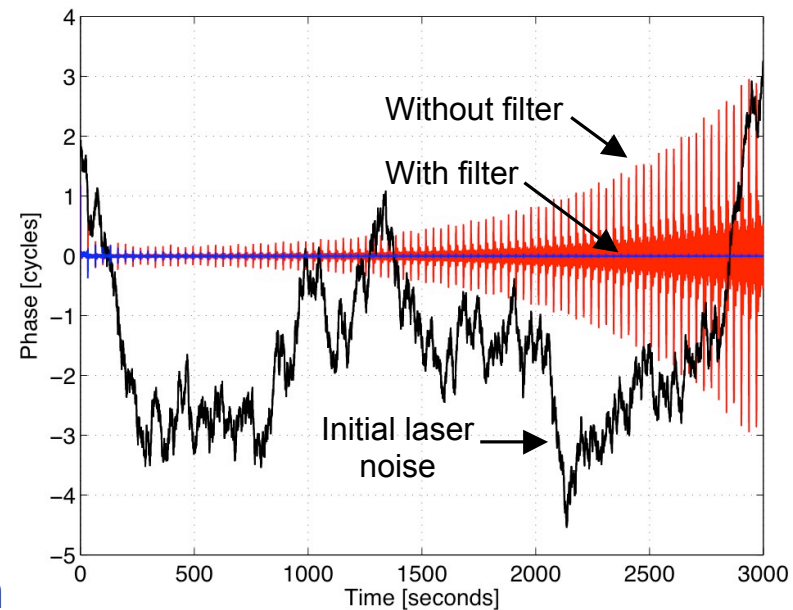
# Direct arm locking instability

- “Simple” direct arm-locking sensor has a control system instability at
$$f = c/(L_1 - L_2) \quad (\sim 3 \text{ Hz for 1\% mismatch}).$$
  - Instability avoided by restricting gain arm locking to  $< 10$  at 3 Hz.

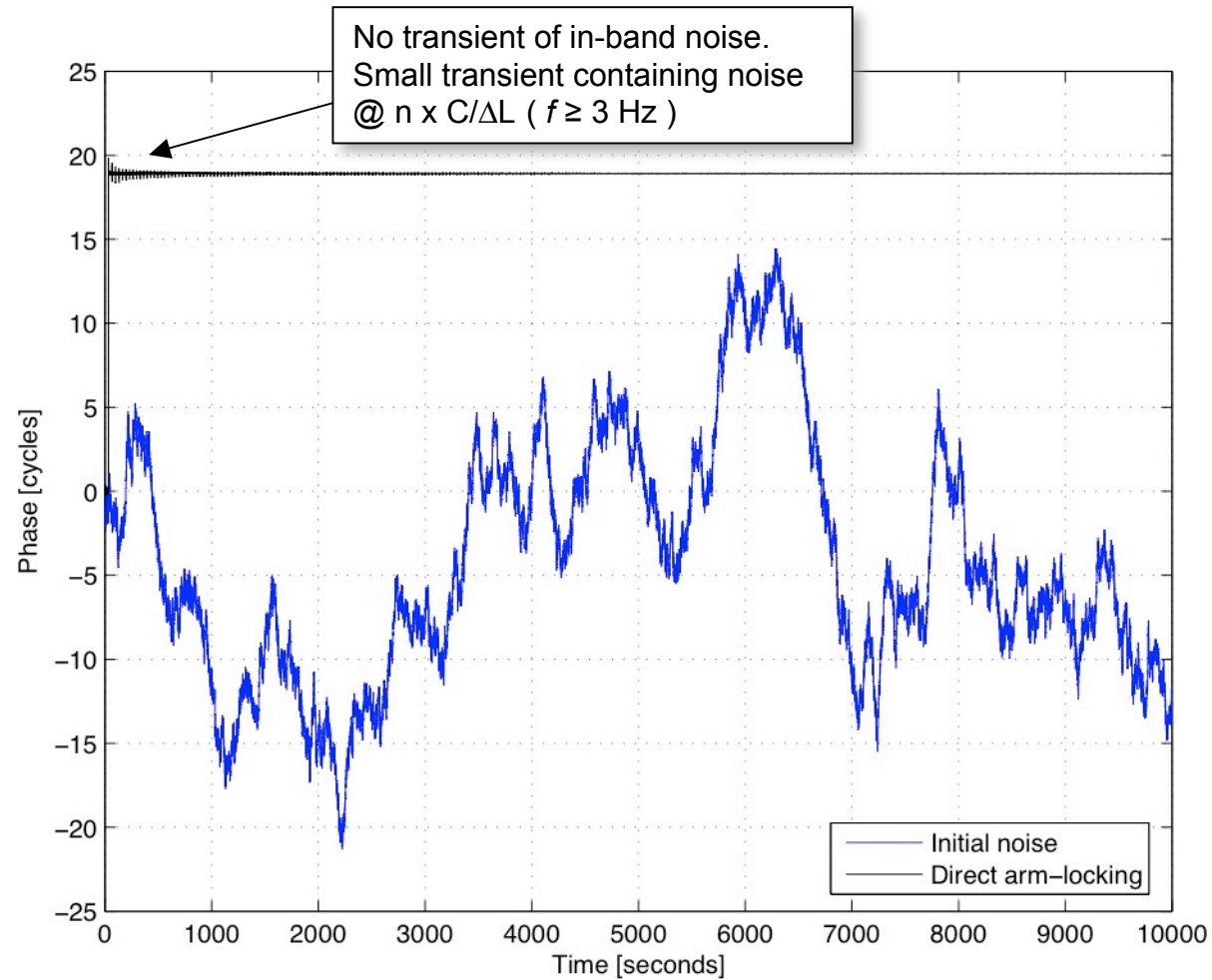


Alternatively, adding a single pole filter in the differential feedback path restores system stability.

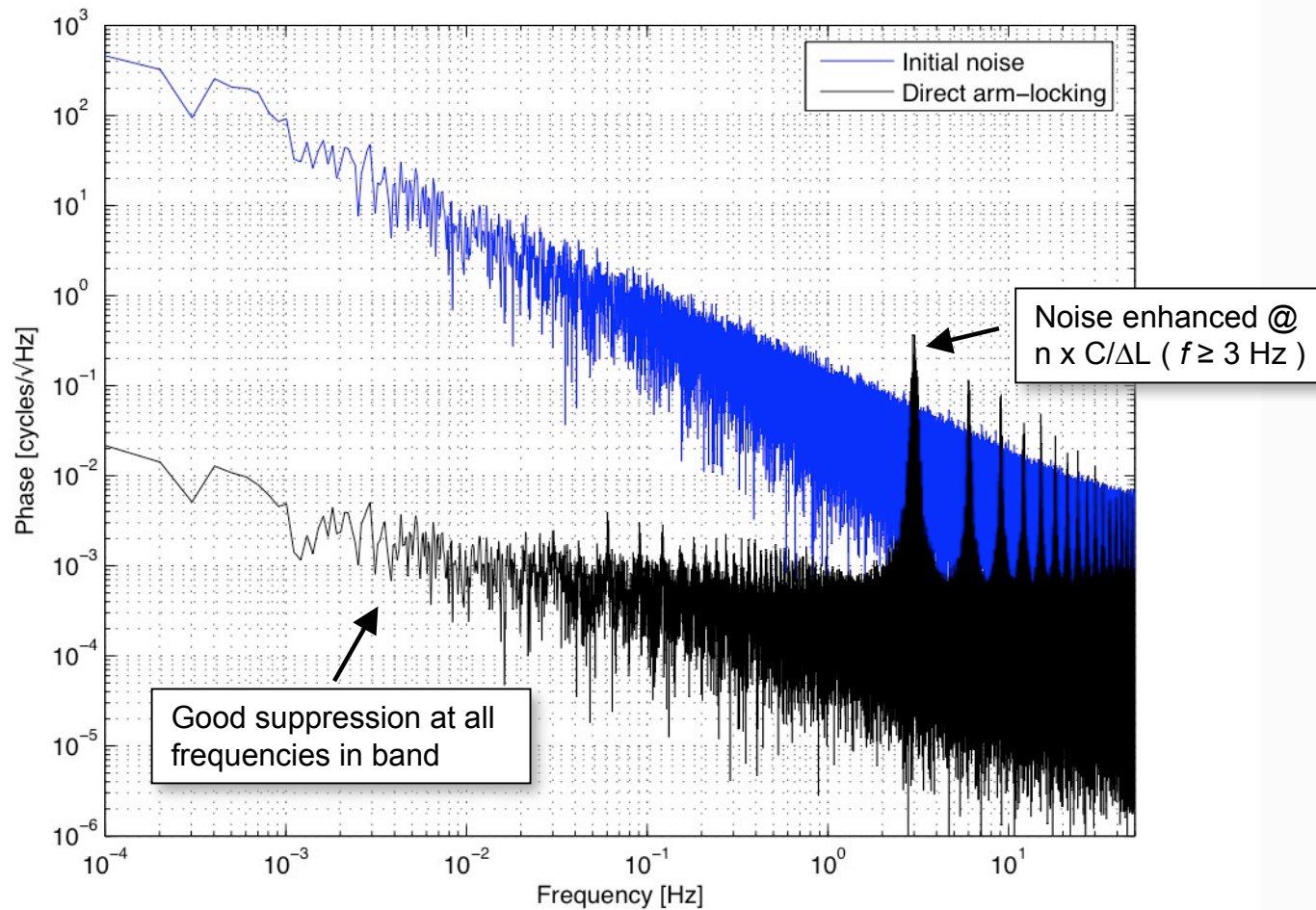
High gain simulation demonstrating instability



# Arm Locking simulation



# Arm-locking simulation



# Arm-locking impact

- Initially arm locking was proposed to reduce risk of TDI failure.
- With a better understanding of TDI and improved arm-locking performance.
  - relaxation of cavity pre-stabilization requirements
  - simplification of flight system, LISA Pathfinder-like stabilization system.



# Time-delay interferometry

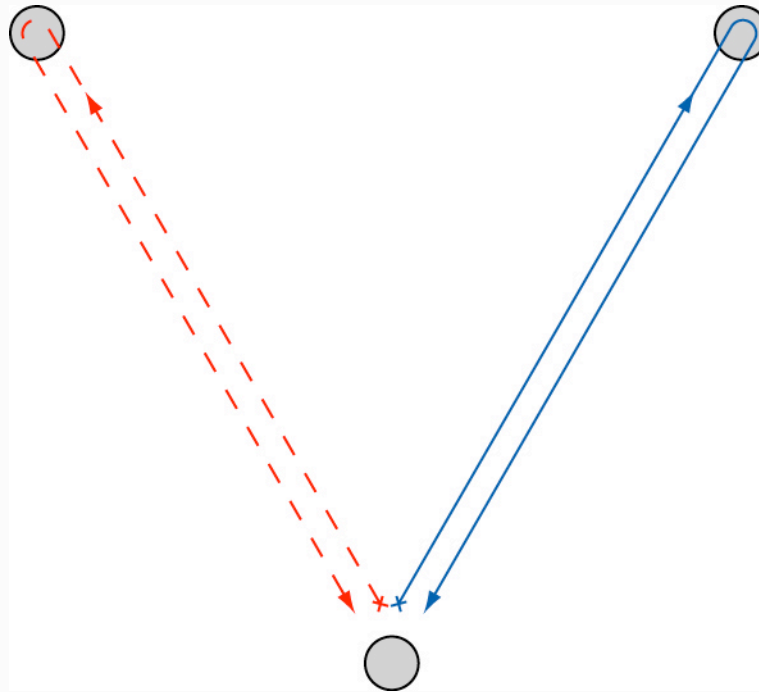
- Time-delay interferometry (TDI) is a post-processing technique for removing:
  - laser frequency noise
  - clock noise (see Bill Klipstein's presentation this afternoon)

See for example M. Tinto, F.B. Estabrook, and J.W. Armstrong, *Time-delay Interferometry for LISA*, Phys. Rev. D, **65**, 082003 (2002).

- TDI synthesizes equal-arm interferometers.
  - Require suppression of laser frequency noise by factor of  $10^8$ .



# Michelson Interferometers

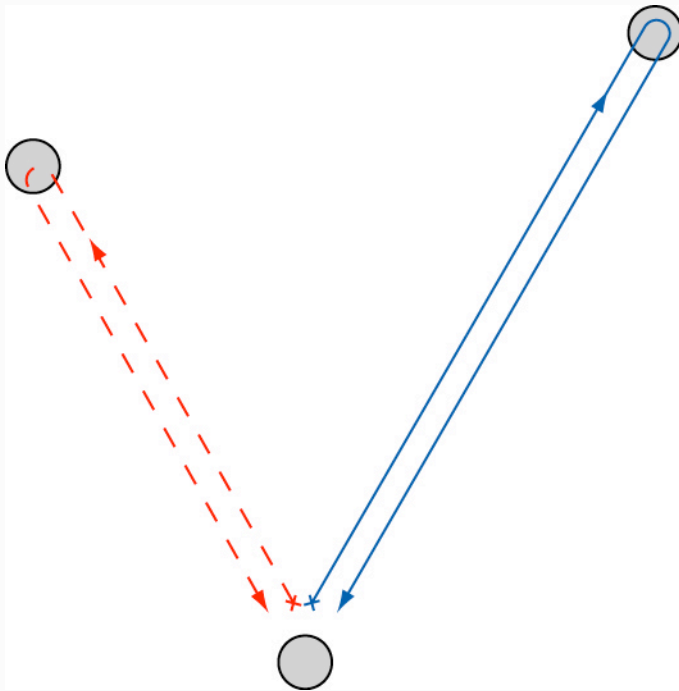


Equal-arm Michelson interferometer

- Insensitive to laser frequency noise

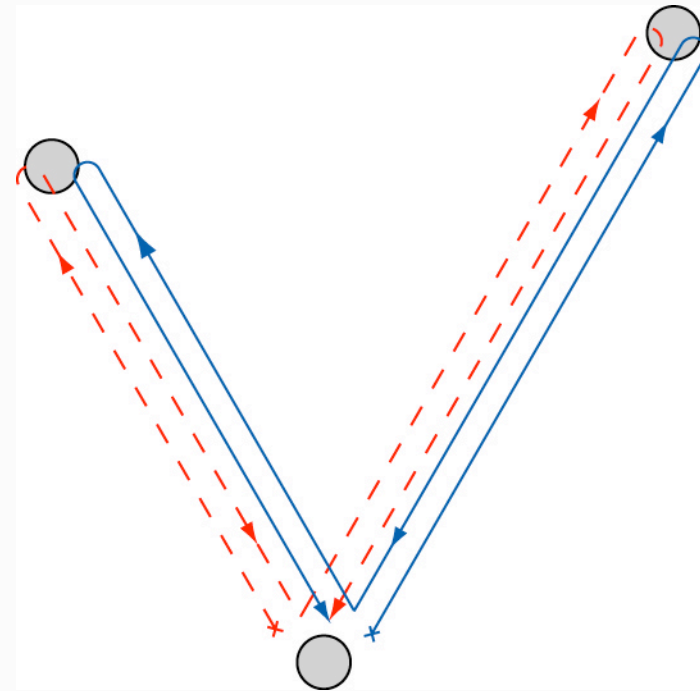


# TDI Michelson



Unequal-arm Michelson interferometer

- Output corrupted by laser frequency noise



Equal-arm (Sagnac) interferometer  
(TDI combination X)

- Output immune to laser frequency noise



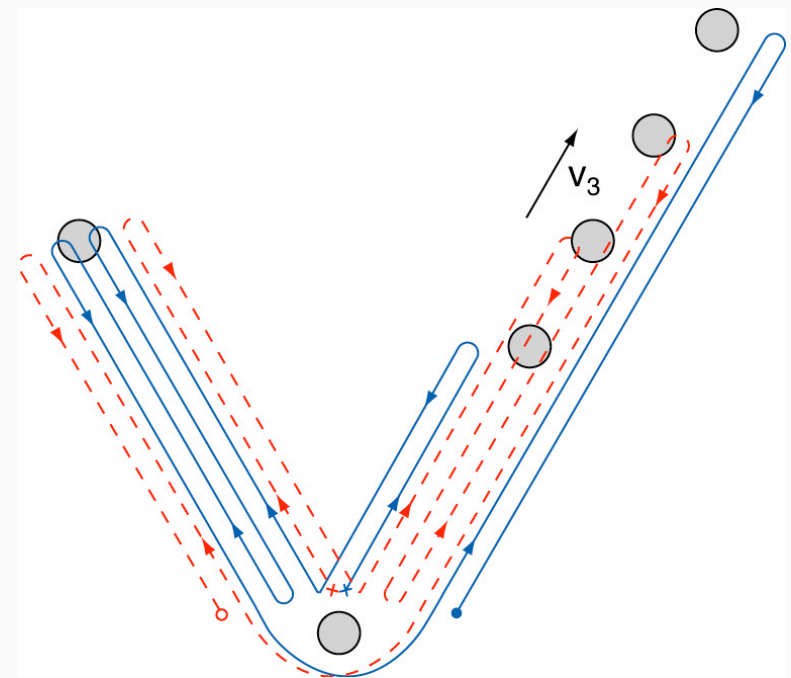
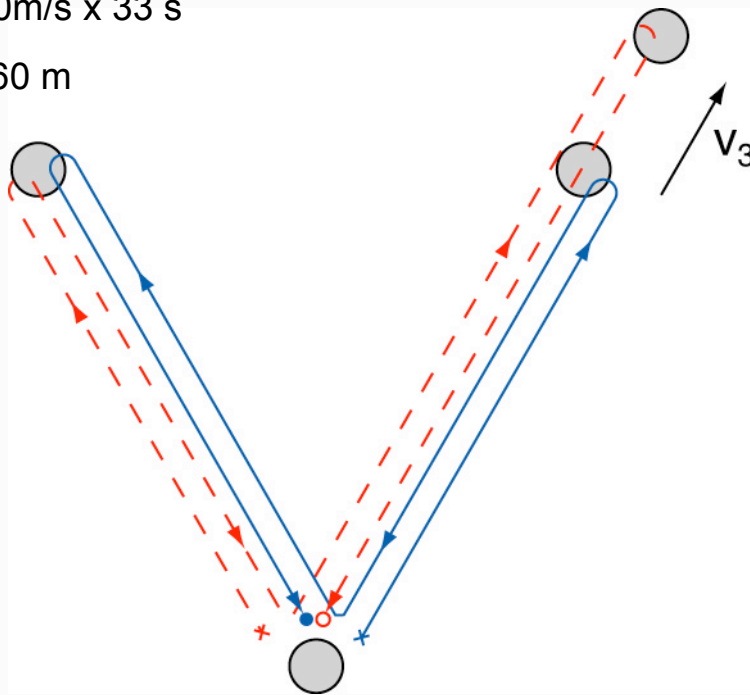
# TDI Michelson with S/C motion

Constant spacecraft velocity introduces an arm length mismatch to the synthesized interferometer.

This arm length mismatch could be as much as

$$\Delta L \sim 20 \text{ m/s} \times 33 \text{ s}$$

$$\sim 660 \text{ m}$$

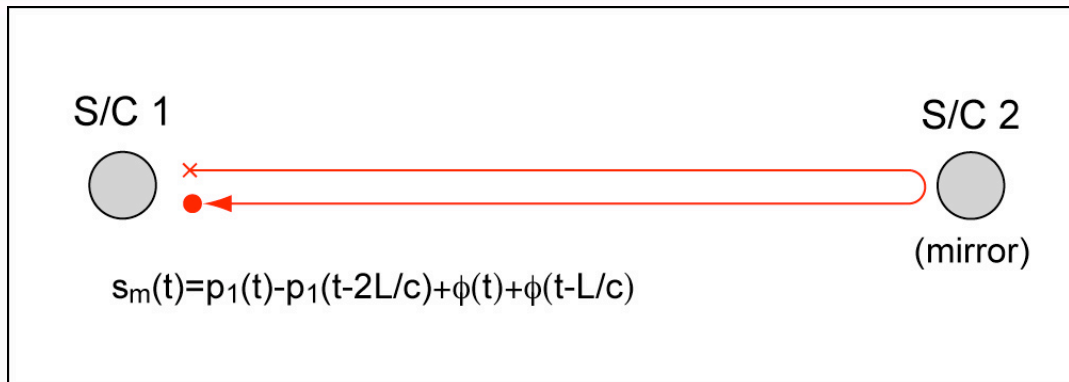


N.J. Cornish and R.W. Hellings, *The effects of orbital motion on LISA time delay interferometry*, Class. Quantum Grav. **20**, 22 4851 (2003)

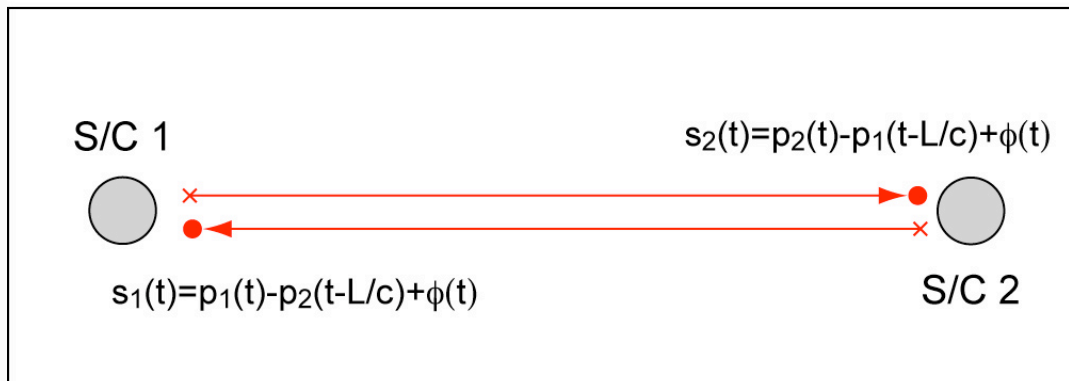
D.A. Shaddock, M. Tinto, F.B. Estabrook, J.W. Armstrong, *Data combinations accounting for spacecraft motion*, Phys. Rev. D **68**, 061303 (2003).



# Synthesizing interferometers?



$p(t)$  laser phase  
 $\phi(t)$  one-way phase shift  
 $s(t)$  phase of interference



Synthesized interferometry combinations have better performance than the equivalent “real” interferometer.

- In real interferometer gravitational wave signal is suppressed or “filtered”.
- In synthesized interferometer all noise sources are also filtered, preserving signal to noise ratio.



# TDI Implementation

- Timing accuracy of phase measurements for time-delay interferometry (TDI) implementation:  $\Delta t < 50$  ns.
- Data transmitted at  $\sim 3$  Hz.
- Baseline approach was to trigger phasemeter measurements at the correct times to 50 ns precision.

M. Tinto, D.A. Shaddock, J Sylvestre, J.W. Armstrong, *Implementation of time-delay interferometry for LISA*, Phys. Rev. D **67**, 122003 (2003).

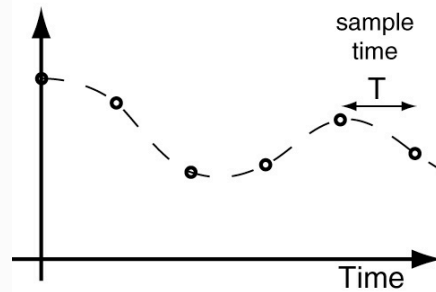
- Alternative approach is to sample at a constant rate and interpolate in post-processing.

D.A. Shaddock, B. Ware, R.E Spero, M. Vallisneri, *Postprocessed time-delay interferometry for LISA*, Phys. Rev. D **70**, 081101R (2004).

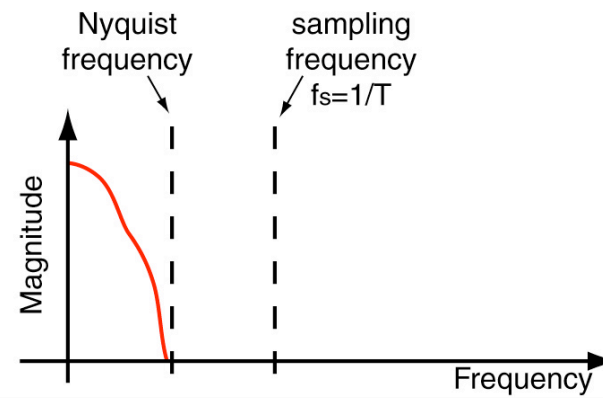


# Interpolation

Time Domain

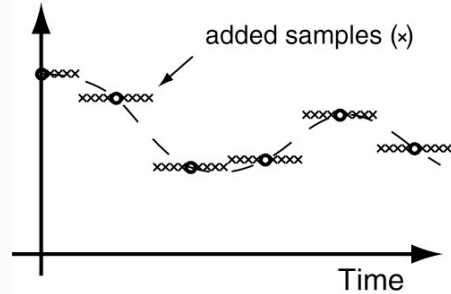
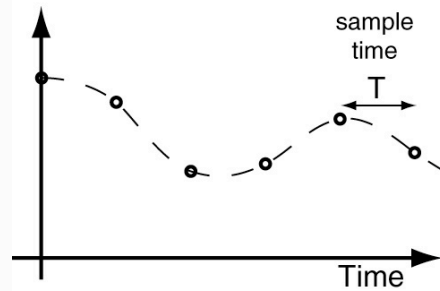


Frequency Domain

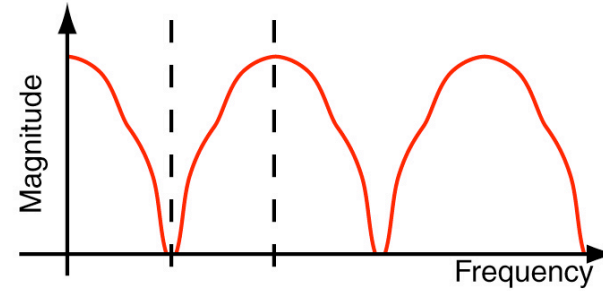
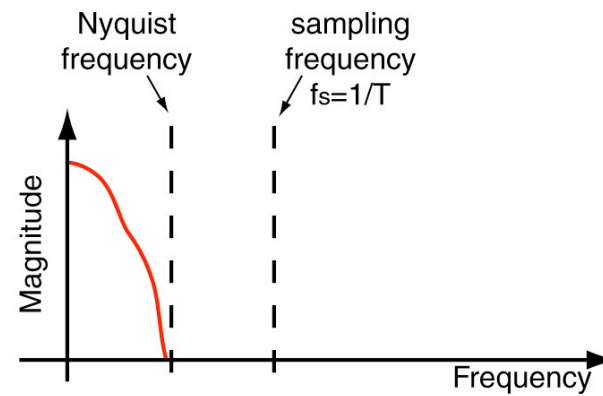


# Interpolation

Time Domain

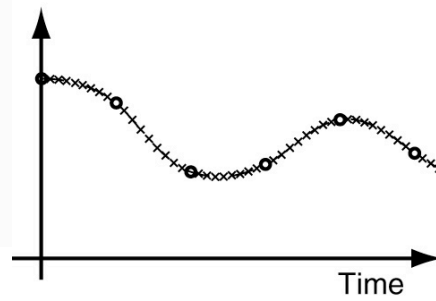
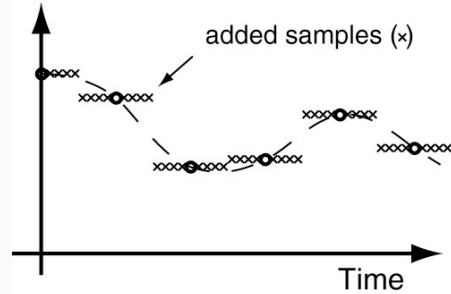
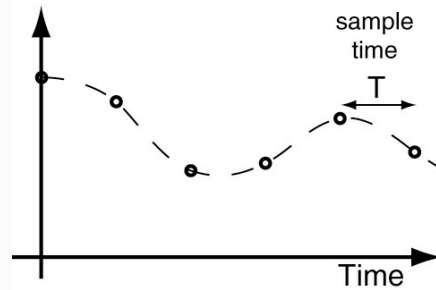


Frequency Domain

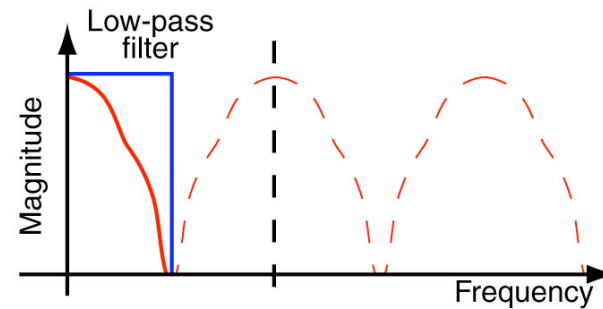
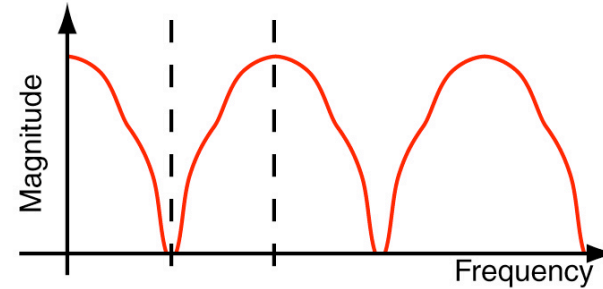
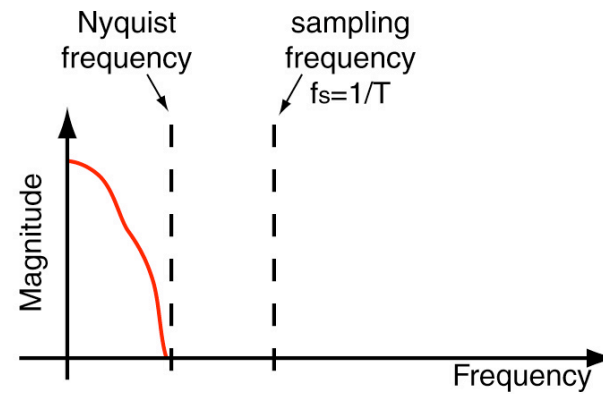


# Interpolation

Time Domain



Frequency Domain

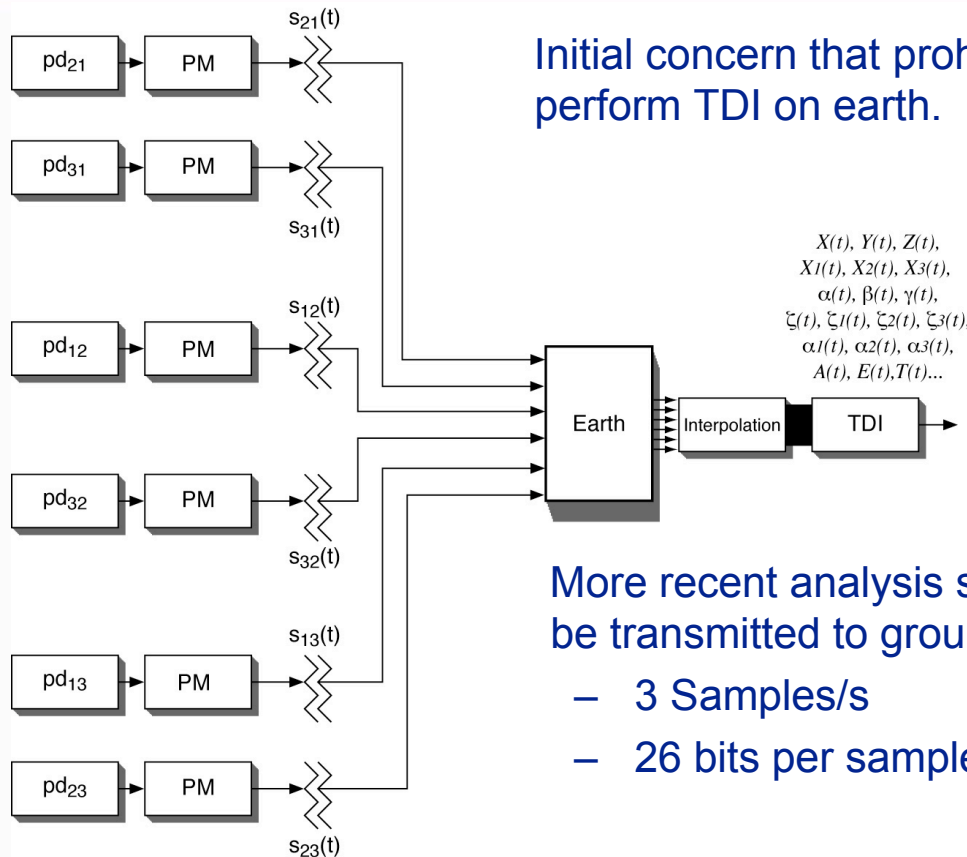


# Post-processed TDI

- With interpolation, delays are implemented in post-processing.
  - Eliminates nanosecond-scale triggering of phase measurements.
  - Phasemeters sample at a constant rate time tagged by local clock.
  - No arm length knowledge needed on-board (by phasemeters, payload computer.)
  - S/C clocks do not need to be synchronized in real-time. A correction can be applied in post-processing.
- Time-delay interferometry combinations can be completely constructed on Earth.
  - Gives scientists access to raw data.
  - Allows flexibility to change algorithms construct all TDI combinations (even combinations not realized until after data is in hand).



# TDI on earth



Science data rate to ground is a small fraction of data rate capability.



# Frequency Noise Simulation

- Simulation of the laser frequency noise processing chain is underway.
- Goal: Demonstrate frequency noise removal
  - Arm-locking simulation (Peter Gath, Hans-Reiner Schulte Astrium).
  - Generate heterodyne phases with appropriate delays (Michele Vallisneri, JPL)
  - Generate heterodyne signals and measure phase with hardware-in-loop (Brent Ware, Shaddock, JPL).
  - Interpolate phase measurements and generate TDI combinations (Vallisneri, JPL).



# Interferometer Gate 1

400 pm/ $\sqrt{\text{Hz}}$  requirement  
13 pm/ $\sqrt{\text{Hz}}$  goal

## Demonstration (TRL3/4) [NASA]

ometry Testbed sensitivity over the interferometry sensitivity  
ing:

1.  $400 \text{ pm}/\sqrt{\text{Hz}} \cdot (1 + (1 \text{ mHz}/f)^4)^{1/2}$ , independent clocks, and independent phasometers, by combining measurements in post-processing.
2. Goal:  $13 \text{ pm}/\sqrt{\text{Hz}} \cdot (1 + (1 \text{ mHz}/f)^4)^{1/2}$ , independent clocks, and independent phasometers, by combining measurements in post-processing.

Using multiple lasers, independent clocks and independent lasers, by combining measurements in post-processing.

Specifically, benchmark the Interferometry Testbed sensitivity over the interferometry sensitivity using the following equation:  
 $400 \text{ pm}/\sqrt{\text{Hz}} \cdot (1 + (1 \text{ mHz}/f)^4)^{1/2}$ , independent clocks, and independent phasometers, by combining measurements in post-processing. Demonstrate that the clock phase fluctuations can be removed by optically transferring the clock phase between measurement points. This gate addresses the Laser/Clock/Phasometer noise (TDI) error budget. As a result of the performance level achieved, NASA will decide whether to continue further activity to improve performance or to stop and accept the present performance.



# Independent Lasers

- LISA interferometry requires excess laser frequency noise be measured with *sub-shot noise limited fidelity* and removed in post-processing.
- In LISA, 6 lasers operating at different frequencies provides multiple heterodyne frequencies.
  - No common mode rejection of many noise effects that are not observed in conventional 2-color heterodyne interferometers.
  - Many new noise sources expected. For example,
    - Frequency noise at  $\pm$  twice the heterodyne frequency.
    - Variable interpolation delay introduces non-linear noise mixing.
    - Non-common aliasing of frequency noise, shot noise and harmonic distortion.
    - Requires careful choice of phasemeter architecture (see Phasemeter presentation on Friday @ 10:50 am.



# Independent Clocks

The clocks on each LISA S/C will be essentially independent except for an optical “link” by phase modulation of the science laser.

Two important errors arise from independent clocks.

1. In-band phase fluctuations of each clock couple into the phase measurement at up to 10,000x higher than target measurement sensitivity.
2. Clock offsets need to be compensated by an interpolation offset to  $\sim 10$  ns precision to adequately cancel laser noise with TDI.

The testbed will demonstrate clock noise cancellation via an optical link (2-8 GHz phase modulation).

Offsets of the independent clocks will be optically measured and compensated by interpolation offsets.



# Representative phase noise

In LISA, phase-locking ensures laser phase noise is correlated below 15 mHz and (on average) uncorrelated above this frequency.

In a testbed with meter-scale arms phase-locking would correlate noise at all frequencies of interest

- Non-representative noise

**If one way measurements are employed between uncorrelated lasers then the arm lengths are irrelevant.**

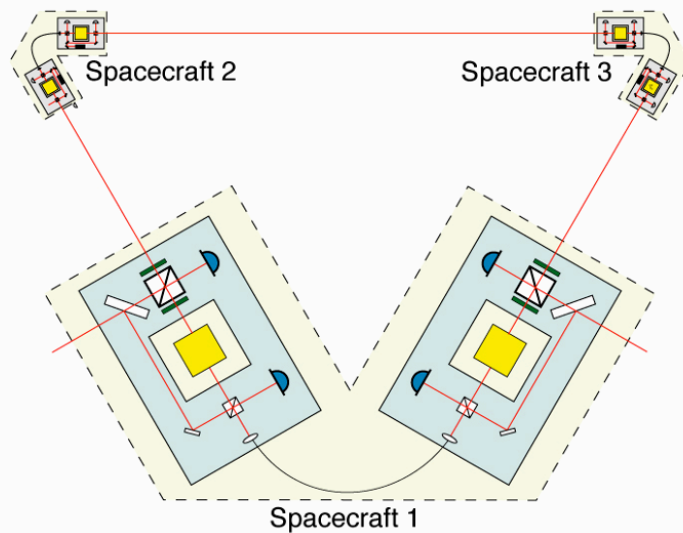
- LISA-like noise statistics are recovered
- Instrument requirements are approximately the same when using one way measurements or phase-locking.

**One way measurements incompatible with arm locking.**

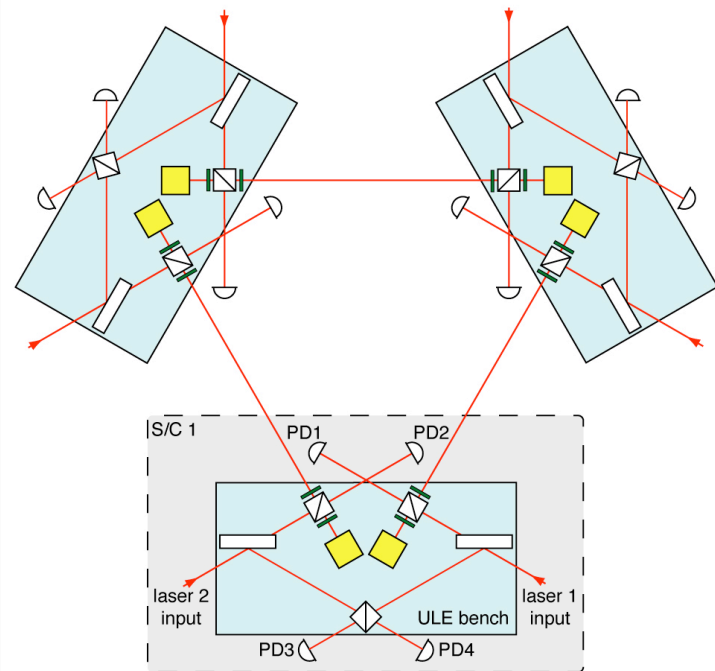
- Impact of arm-locking on instrument requirements can be simulated by controlling relative stability of independent lasers.
- Arm-locking tests should be performed elsewhere (e.g. UF Testbed)



# Testbed/LISA Comparison



Simplified LISA schematic



Testbed schematic



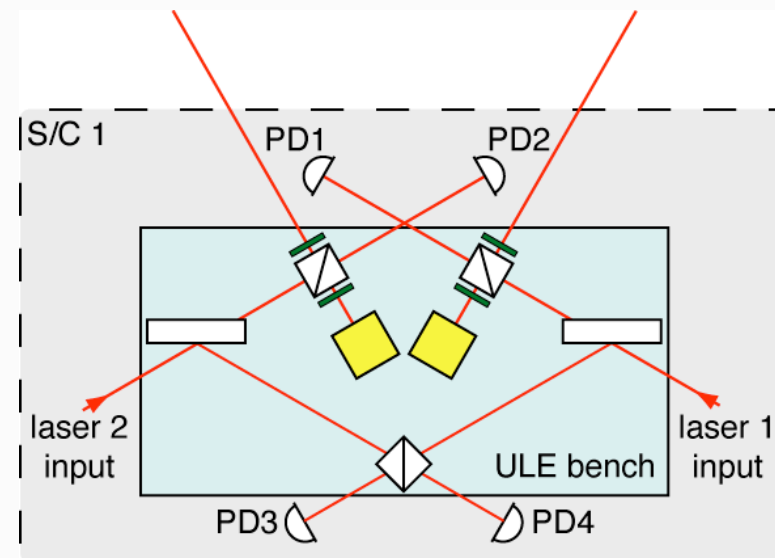
# Simplified bench topology

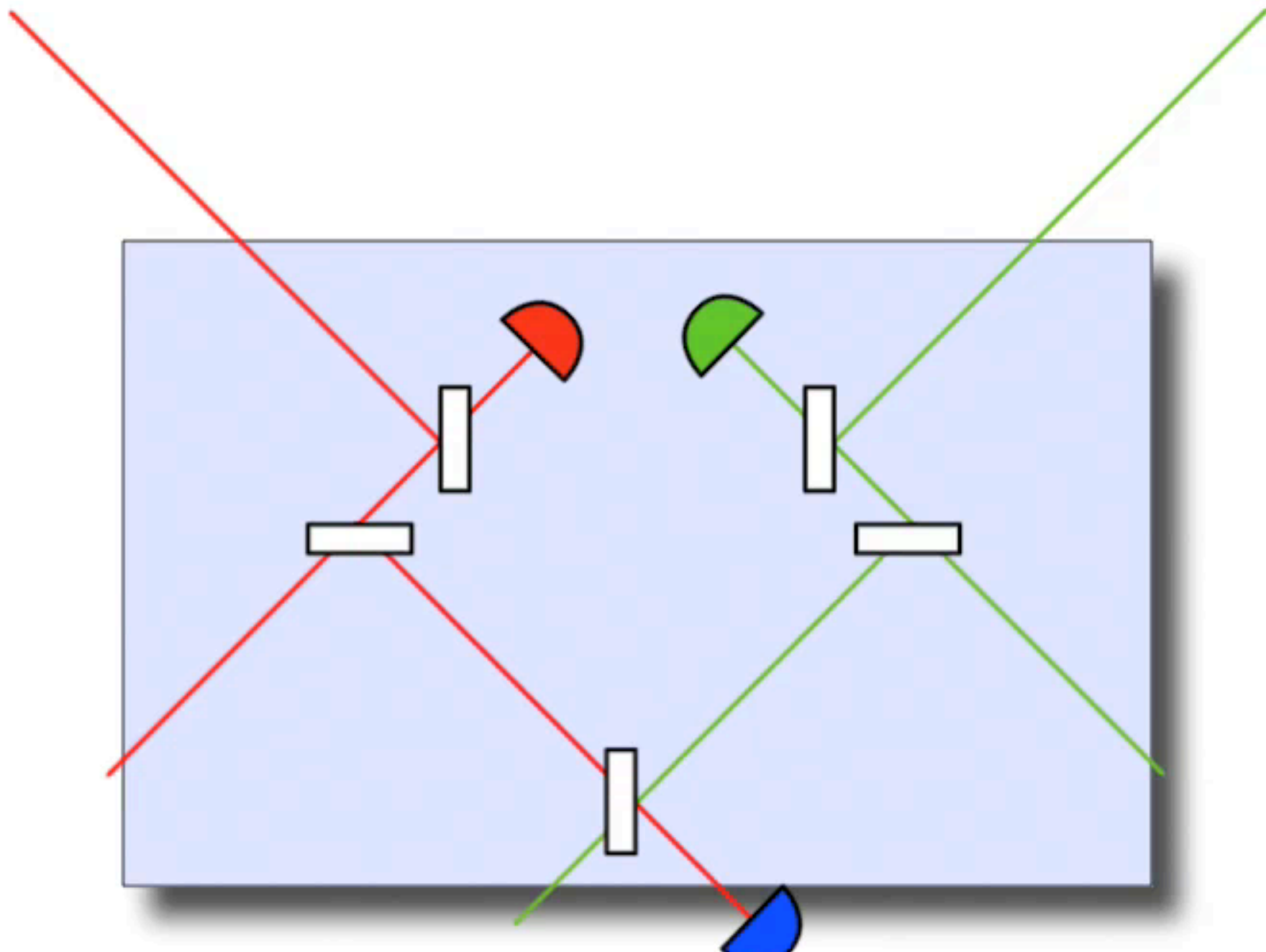
Proof mass is bonded directly to bench.

- Bench-PM motion enters in the same way as frequency noise.
- Frequency noise is  $\sim 10^5$  times larger than bench-PM motion.

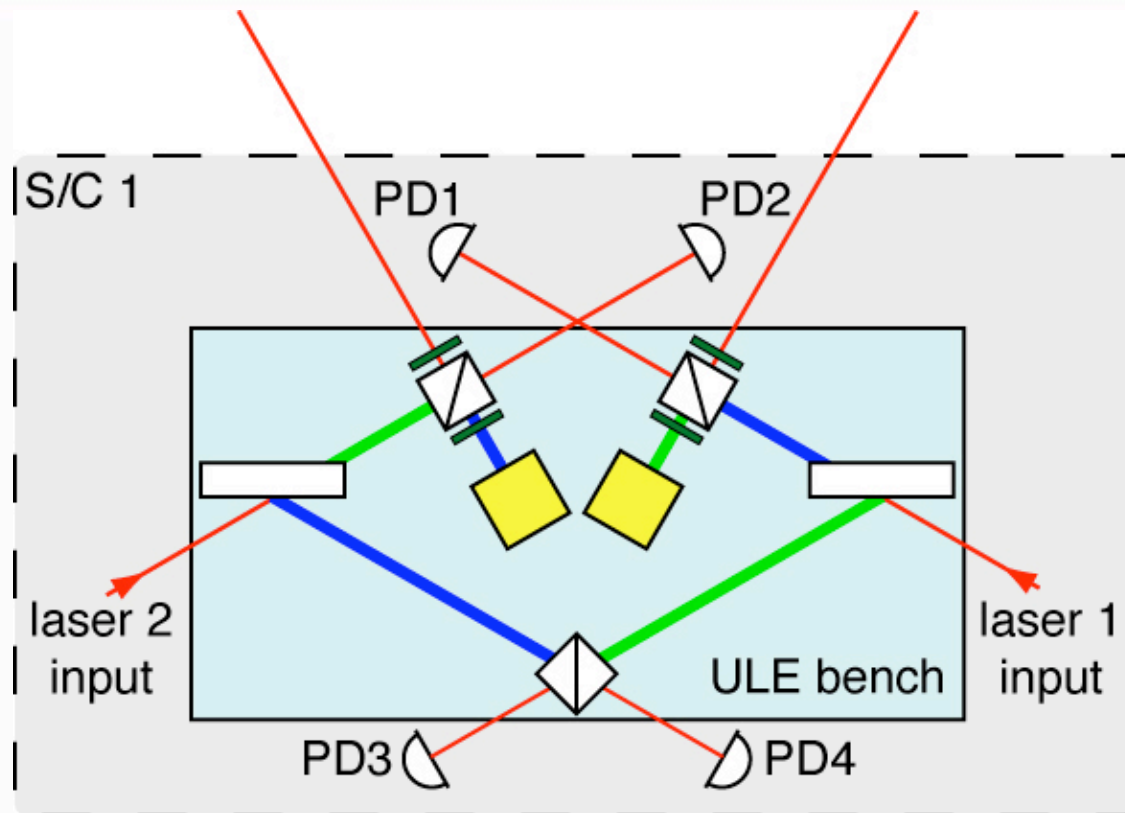
Simplified aft-interferometry.

- No reflection from back of PM
- No additional interferometer for individual bench-PM motion.
- Does not include aft-fiber noise.
- No strap down or frequency swapping



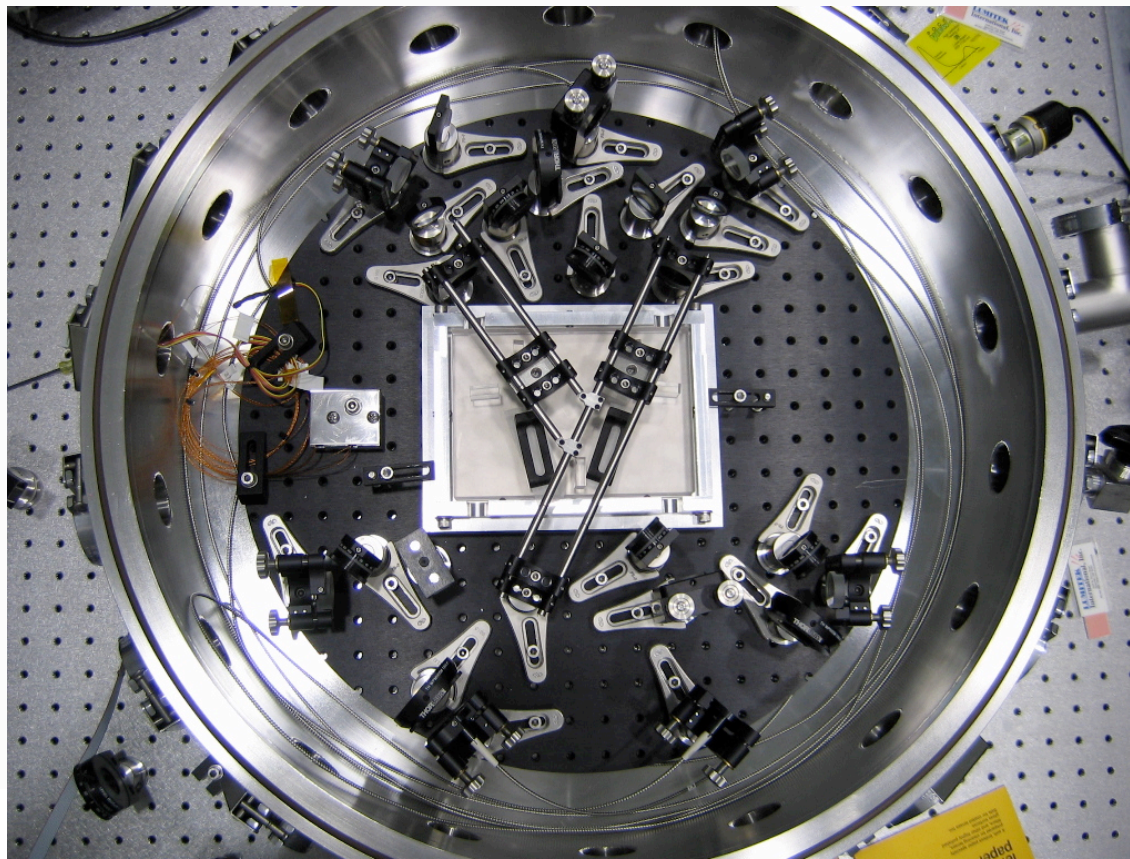


# Sensitive optical paths

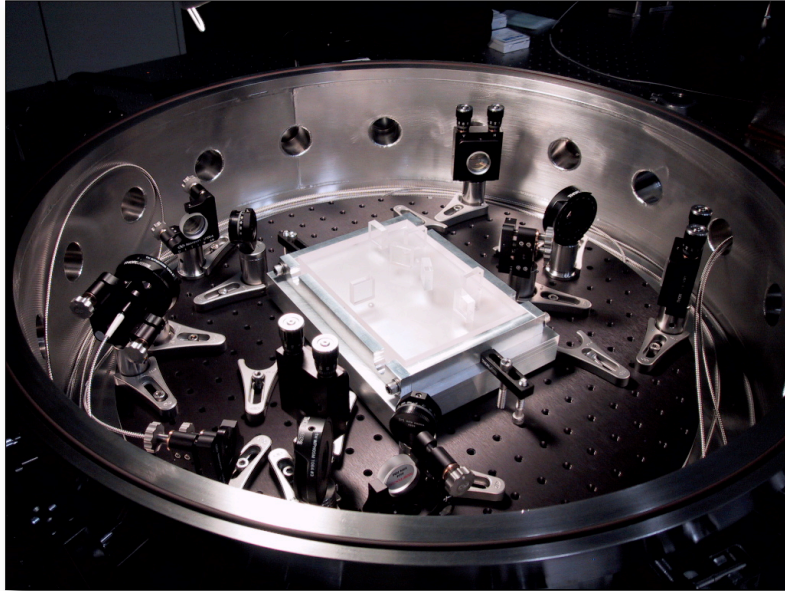


By ensuring the interferometer output is sensitive optical path we avoid artificial, purely algebraic cancellation of laser noise.

People: Brent Ware, Bob Spero, Bill Klipstein,  
Akiko Hirai and Rachel Cruz.

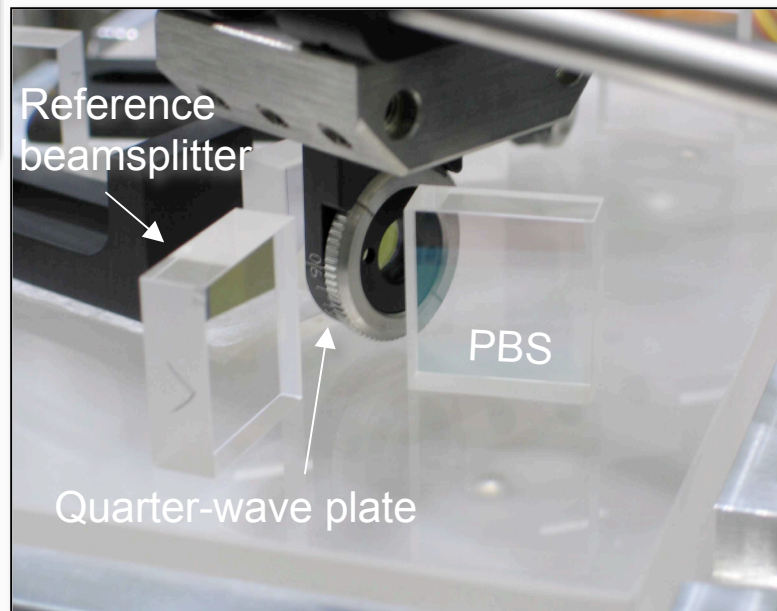


# ULE Construction

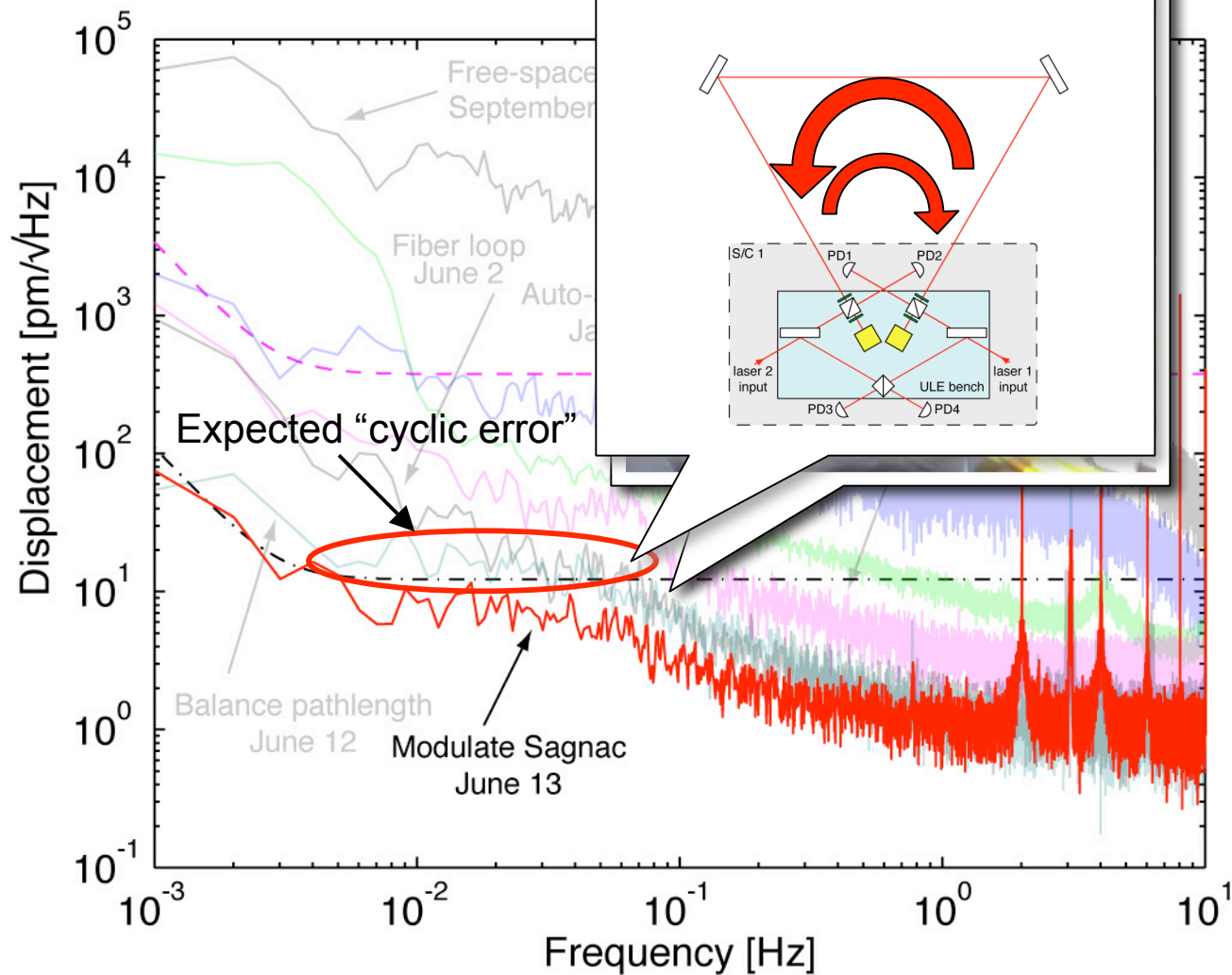


- Polarization leakage field readout.
- Plate polarizing beamsplitters
- Wave-plates suspended from above

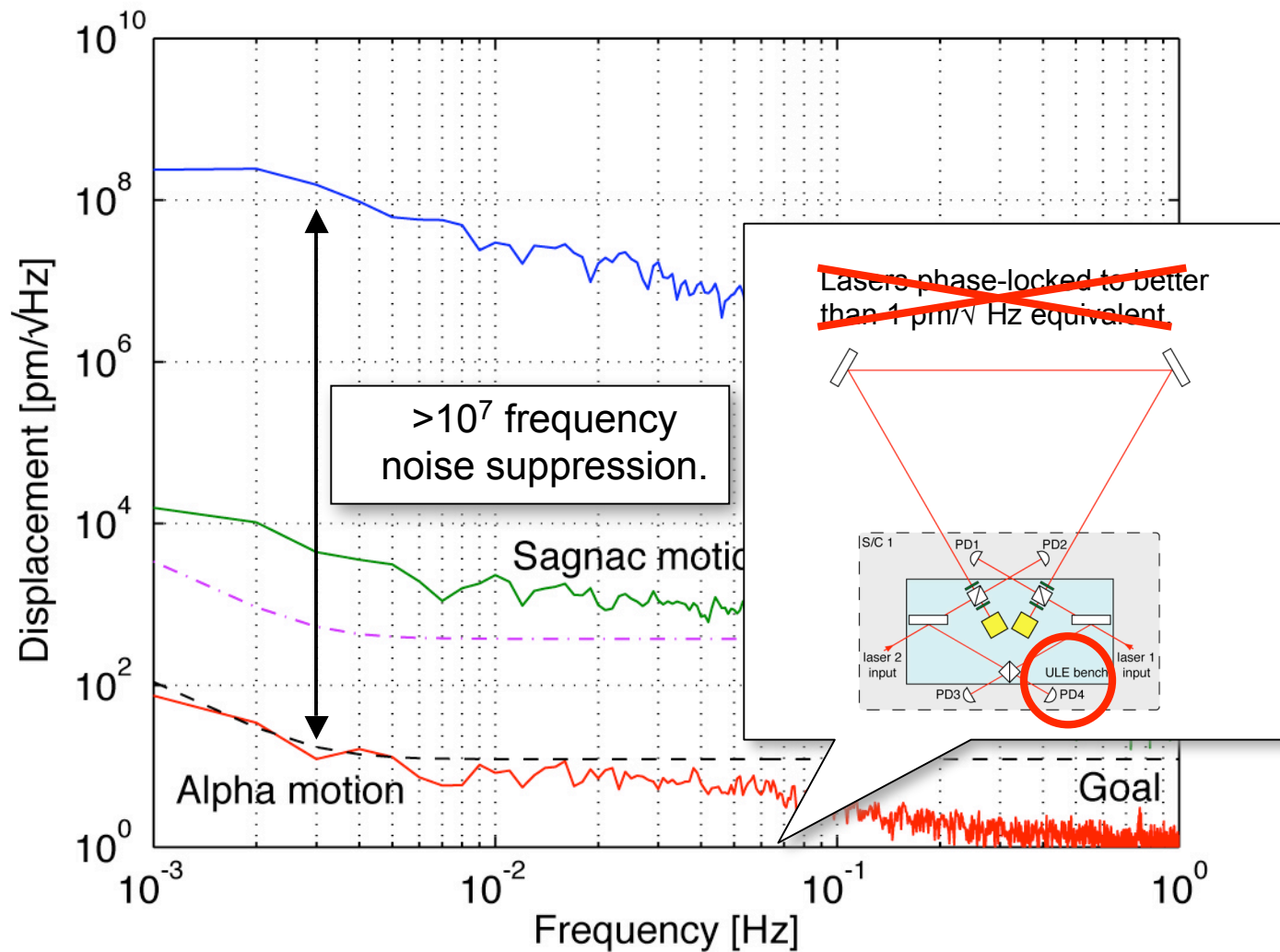
- Fused silica optics optically contacted to ULE bench.
- Optical alignment measured using optical cavity based system.



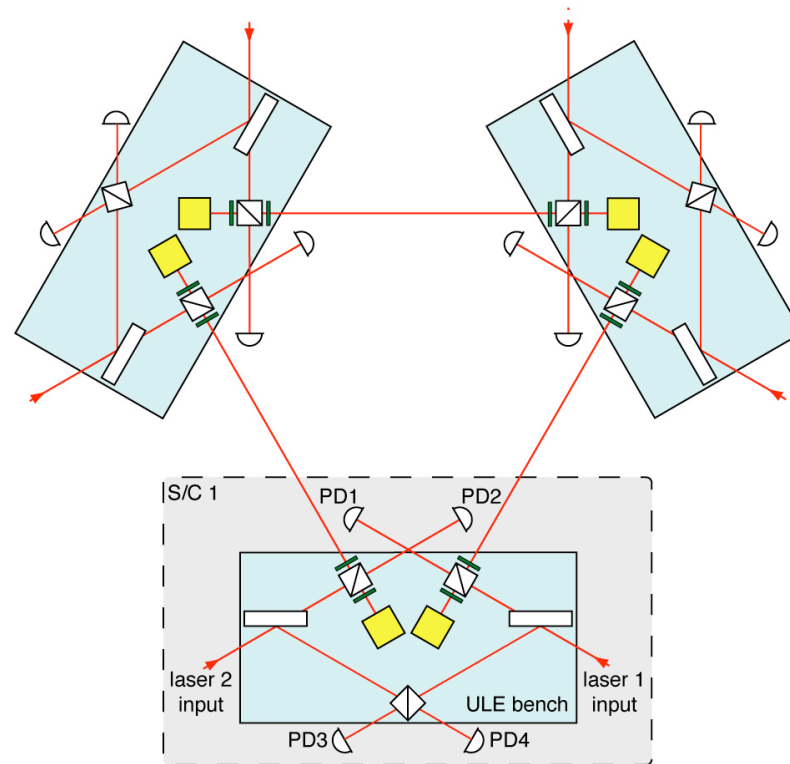
# Interferometry Testbed



# Noise cancellation



# What's next for test bed?



“The team at JPL has just completed the test bed development with a single laser, with hardware equivalent to single spacecraft. They still need to build a system with at least 3 lasers and different clock systems to simulate 3 spacecraft system. This will demonstrate TRL 6.”

Sachidananda Babu  
GSFC

LISA Technology Review, December 2006



# Summary

Improved arm-locking implementation.

- More robust control
- Improved noise suppression
- No startup transients or initialization procedure.

Simplified Time-delay interferometry implementation adopted by project.

- All TDI combinations constructed on the ground.
- Retain raw data.
- No real-time ranging, clock sync and inter-SC comm required.

Tests of TDI underway

- High fidelity, hardware-in-loop simulation work in progress.
- First stage of interferometry testbed now meets sensitivity requirement and goal.
- $10^7$  noise suppression demonstrated, albeit with single heterodyne frequency.

